Everything you need to know

Numerical Weather Prediction

in about 100 minutes

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What is NWP?

- A quantitative future forecast of weather (or climate) based on a model or a set of model or a set of model solutions to predict temperature, wind, rain, snow, hail, etc. over a prescribed domain
- Forecast is created from a set of PDE's and other process equations that describe the dynamic and thermodynamic processes in the earths atmosphere
- The domain and horizontal and vertical grid structure and domain is a fundamental choice which heavily impacts the equation set and model performance
- PDE's are discretized using a set of basis functions appropriate (more or less) to the domain of interest. Not all scales of motion & processes are represented
- Unresolved processes need to be "parameterized" - cannot ignore them
- PDEs need initial and boundary conditions
- These are marched forward in time to represent the "weather"

All NWP forecasts....

- Omit some set of processes
- Estimate others
- Have temporal and spatial resolution that are 100s if not 1000s of times coarser than the scales and effects we are trying to represent.
- Numerical approximations PDEs have systematic errors
- Parameterizations are gross
- Don’t know the initial state well enough?

In many ways...

- NWP models reflect our “best” understanding of the motions and processes in the atmosphere
- They also reflect our limits of knowledge and our inherent tendency to be biased
- Observations also limit prediction
- We are always improving them (or trying to) despite these challenges

What is NWP?

- A set of PDE's and other equations that describe the dynamic and thermodynamic processes in the earths atmosphere
  - Equations
  - Numerical approximations
  - Parameterizations
  - Domains
  - Initial and boundary conditions

Equations used

- Conservation of momentum
  - 3 equations
- Conservation of mass
  - 1 for air (continuity)
  - 1 for water
- Conservation of energy
  - 1 equation for first law of thermodynamics
- Relationship between density, pressure and temperature
  - Eq. of State....

Almost every model uses a slightly different set of equations

- Why?
  - Application to different parts of the world
  - Focus on different atmospheric processes
  - Application to different time and spatial scales
  - Ambiguity and uncertainty in formulations
  - Tailoring to different uses
  - History and model developer(s) heritage....
- **What are differences between these NWP requirements?**
  - global prediction versus climate prediction
  - regional prediction versus global prediction
  - storm-scale versus regional prediction

What do the PDEs look like?

Equations of motion (ECWMF model)

East-west wind

$$\frac{\partial U}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial U}{\partial \lambda} + v\cos\theta\frac{\partial U}{\partial \theta}\right\} + \dot{\eta}\frac{\partial U}{\partial \eta}$$

$$(-fv) + \frac{1}{a}\left\{\frac{\partial \phi}{\partial \lambda} + R_{\mathrm{dry}}T_{\mathrm{v}}\frac{\partial}{\partial \lambda}(\ln \boldsymbol{p})\right\} = \underline{P_U + K_U}$$

North-south wind

$$\frac{\partial V}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial V}{\partial \lambda} + V\cos\theta\frac{\partial V}{\partial \theta} + \sin\theta(U^2 + V^2)\right\} + \dot{\eta}\frac{\partial V}{\partial \eta}$$

$$+ fU + \frac{\cos\theta}{a}\left\{\frac{\partial \phi}{\partial \theta} + R_{\mathrm{dry}}T_{\mathrm{v}}\frac{\partial}{\partial \theta}(\ln \boldsymbol{p})\right\} = \underline{P_V + K_V}$$

Temperature

$$\frac{\partial T}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial T}{\partial \theta} + V\cos\theta\frac{\partial T}{\partial \theta}\right\} + \dot{\eta}\frac{\partial T}{\partial \eta} - \frac{\kappa T_{\mathrm{v}}\omega}{(1 + (\delta - 1)\boldsymbol{q})\boldsymbol{p}} = \underline{P_T + K_T}$$

Humidity

$$\frac{\partial \boldsymbol{q}}{\partial t} = \frac{1}{a\cos^2\theta}\left\{U\frac{\partial \boldsymbol{q}}{\partial \lambda} + V\cos\theta\frac{\partial \boldsymbol{q}}{\partial \theta}\right\} = \eta\frac{\partial \boldsymbol{q}}{\partial \eta} = \underline{P_q + K_q}$$

Continuity of mass

$$\frac{\partial}{\partial t}\left(\frac{\partial \boldsymbol{p}}{\partial \eta}\right) + \nabla.\left(\mathbf{v}_{\mathrm{H}}\frac{\partial \boldsymbol{p}}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial \boldsymbol{p}}{\partial \eta}\right) = 0$$

Surface pressure

$$\frac{\partial \boldsymbol{p}_{\mathrm{surf}}}{\partial t} = -\int_0^1 \nabla.\left(\mathbf{v}_{\mathrm{H}}\frac{\partial \boldsymbol{p}}{\partial \eta}\right)d\eta$$

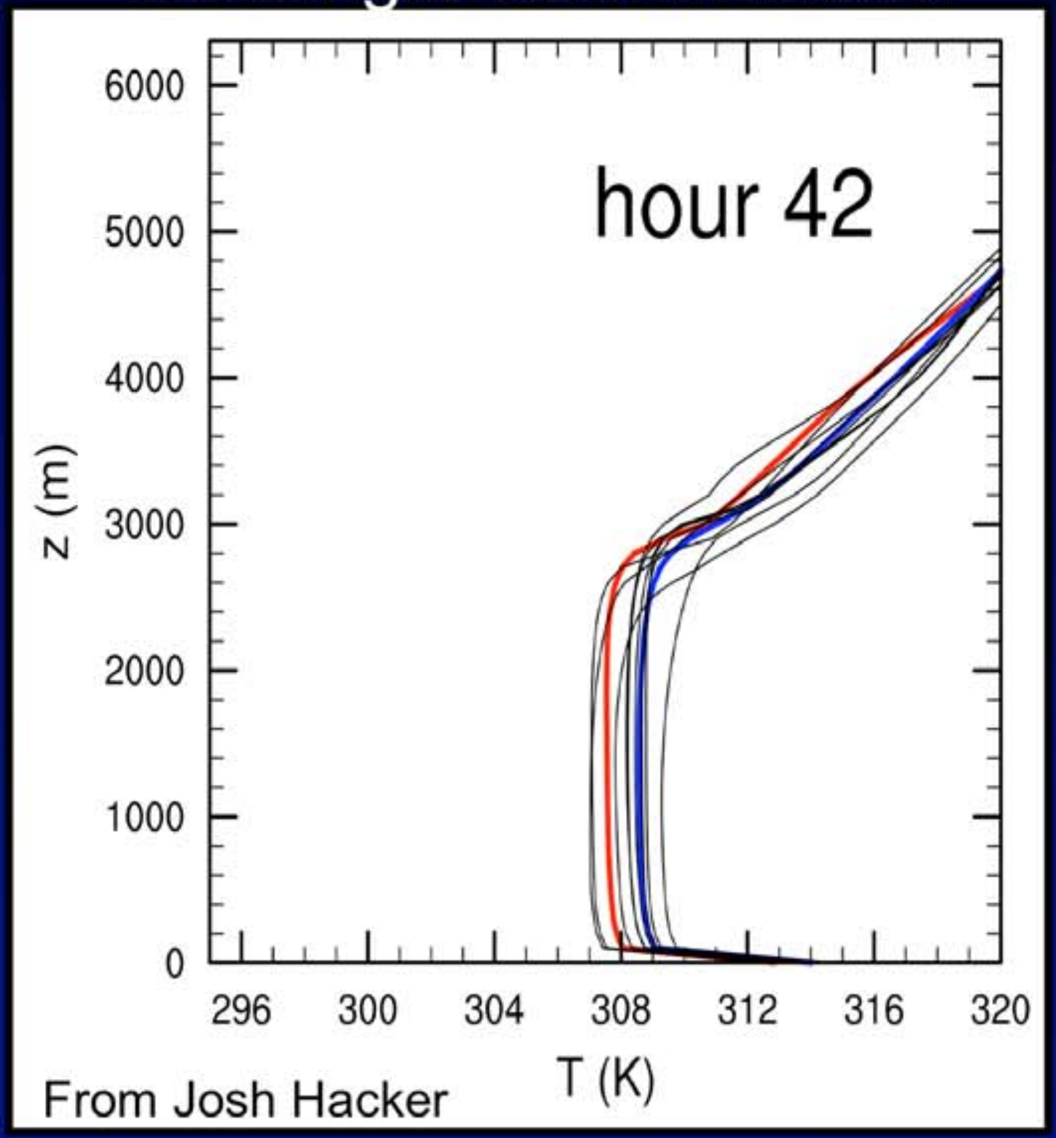
Domains

- Number of dimensions
- Degree and kind of structure
- Shape
- Vertical coordinate
- Resolution

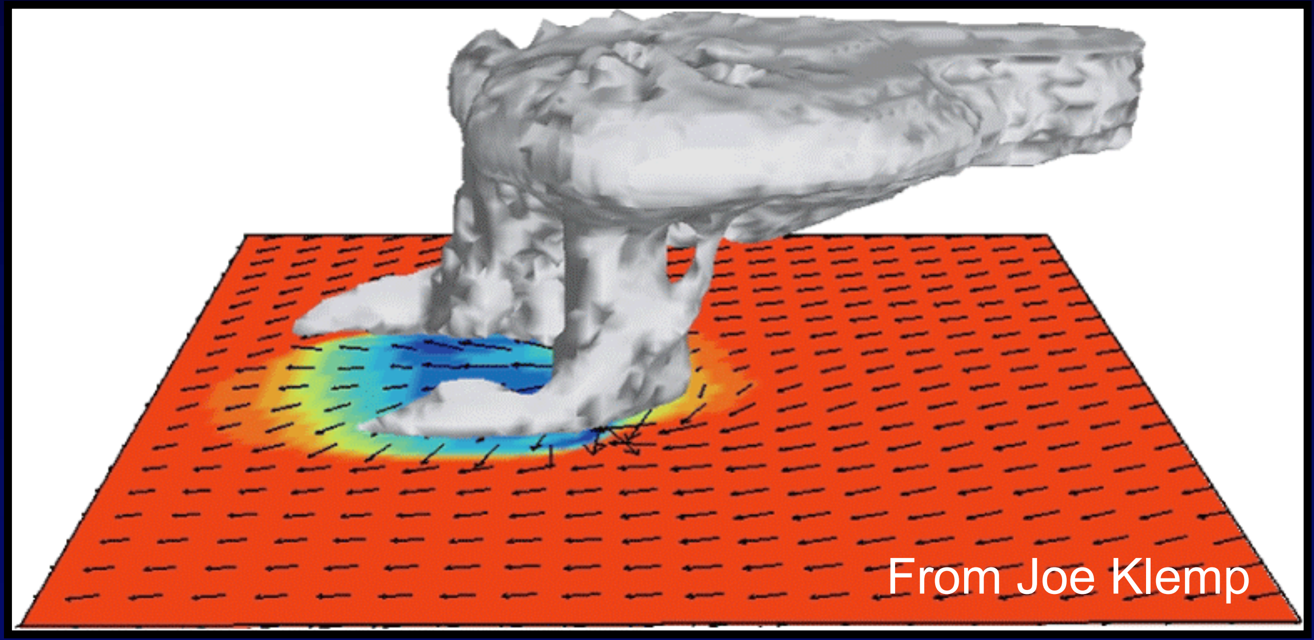
Domains

- Number of dimensions

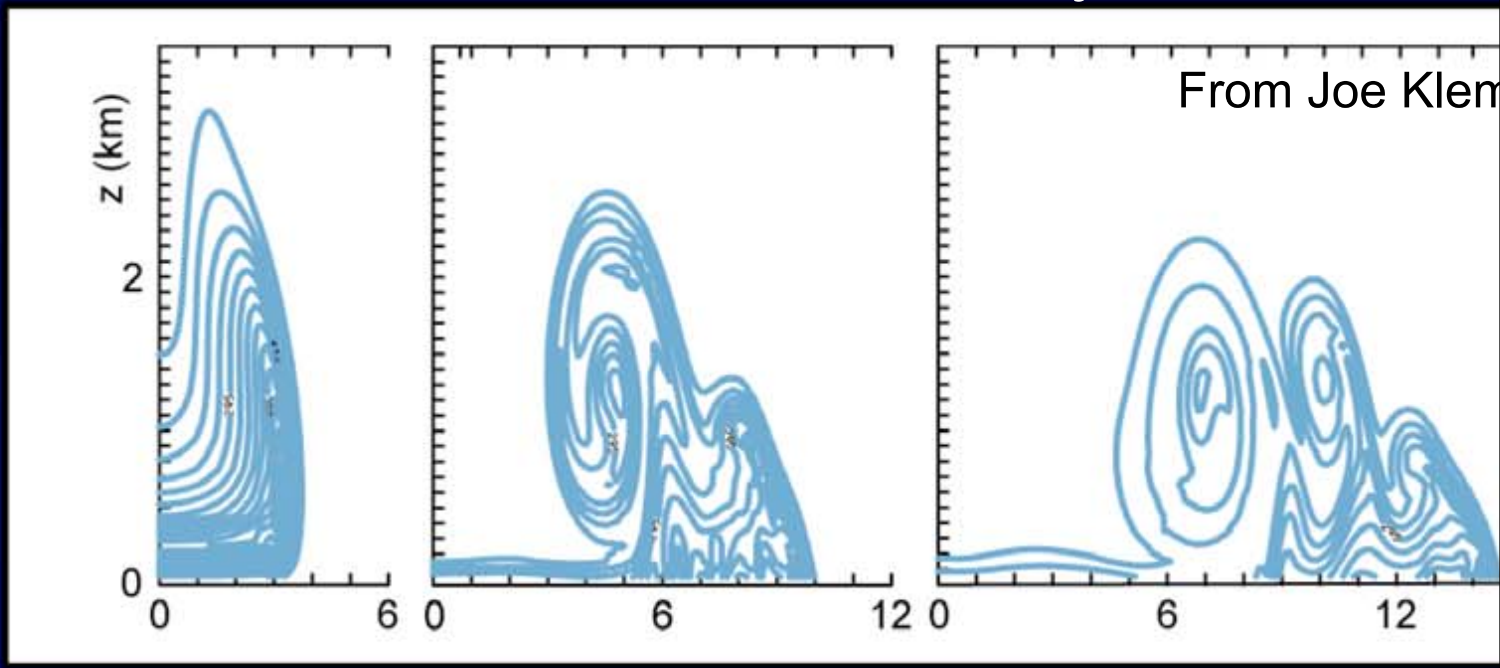
1D: Single-column model



3D: Simulation of thunderstorm



2D: Simulation of density current

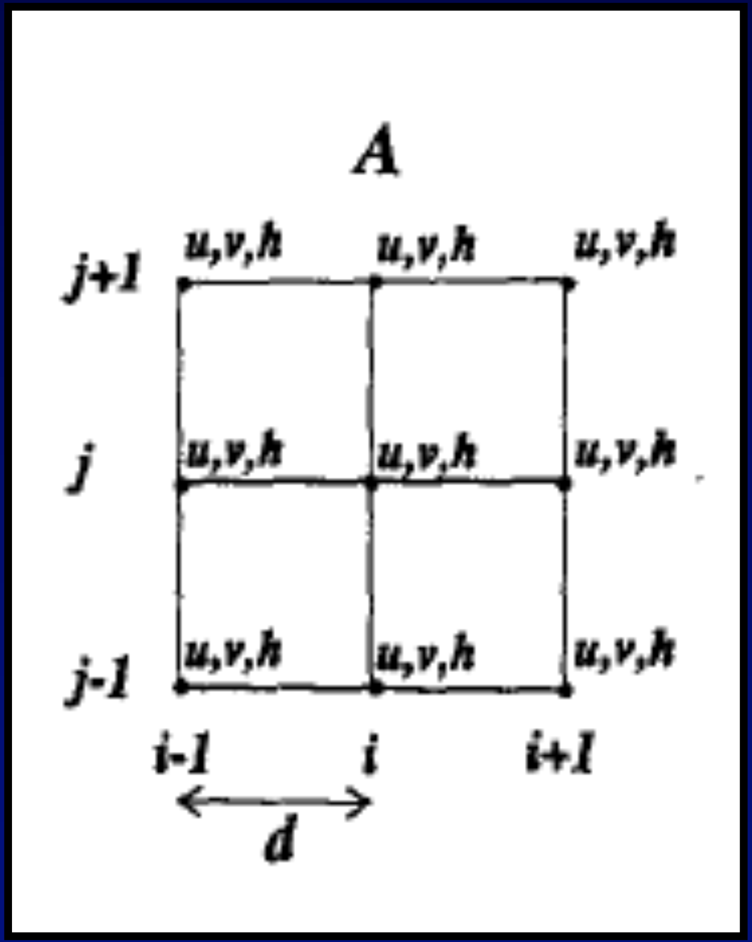


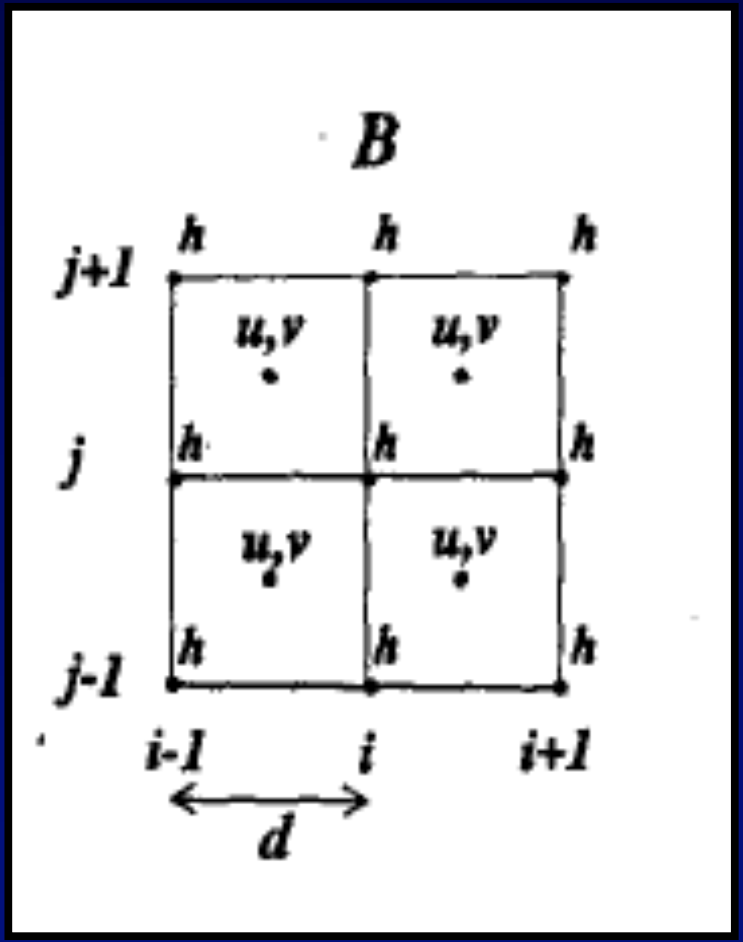
Domains

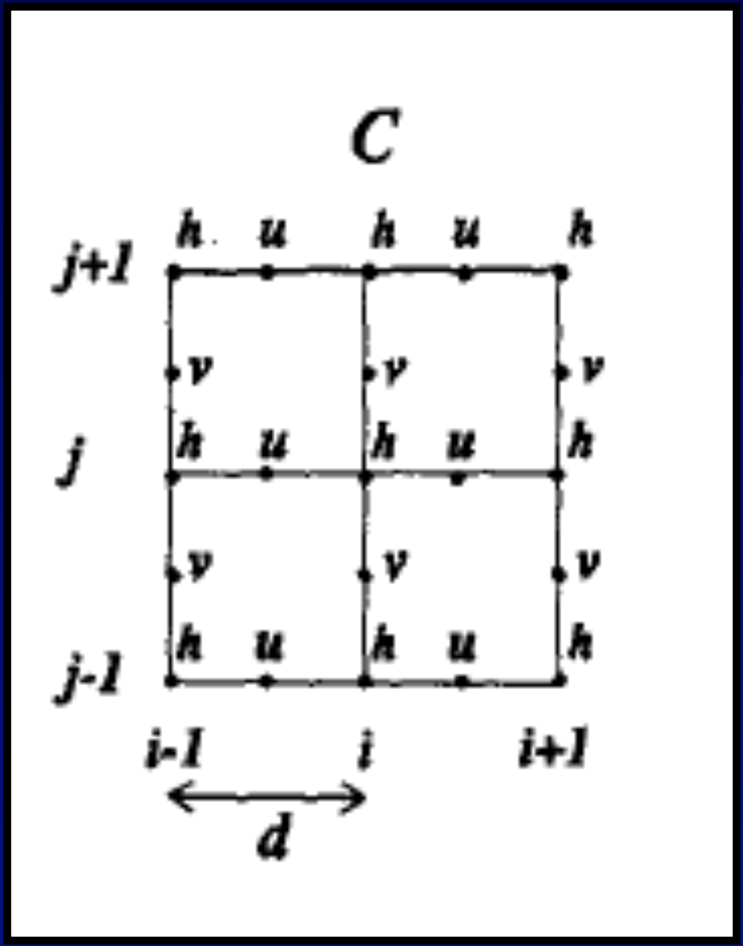
- Degree and kind of structure

MM5 and others

WRF and others





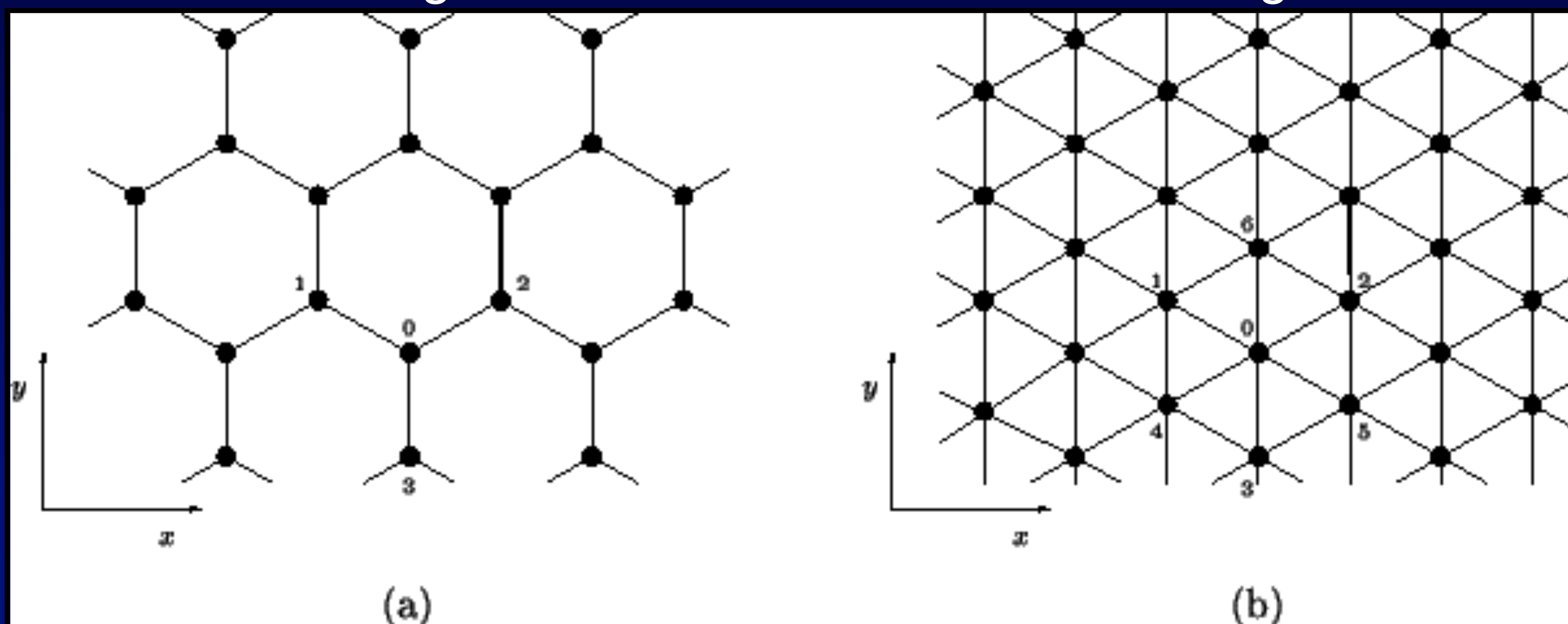


From Randall (1994)

Domains

- Degree and kind of structure

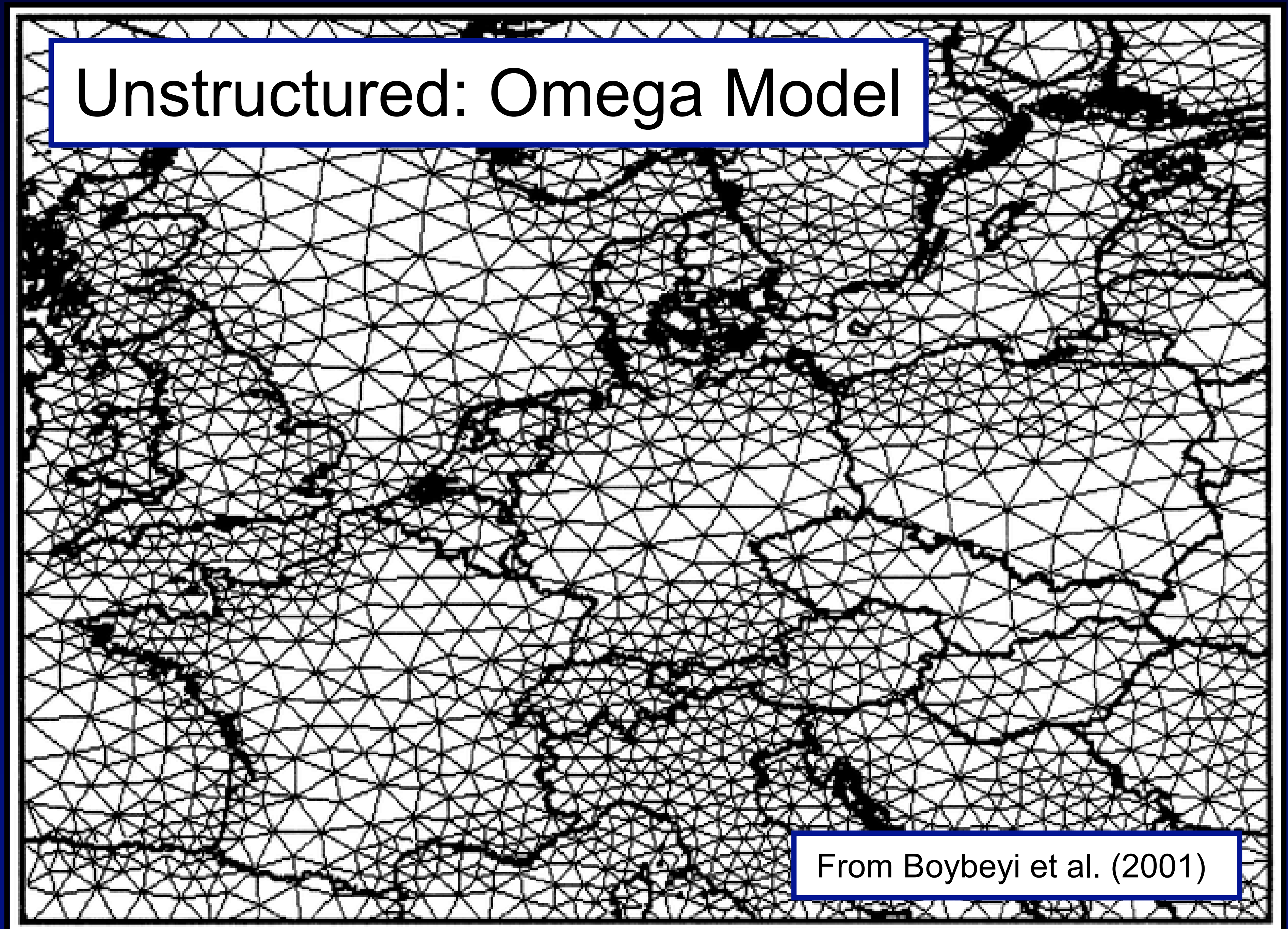
Hexagonal Triangular



From ccrma.standford.edu/~bilbao

Domains

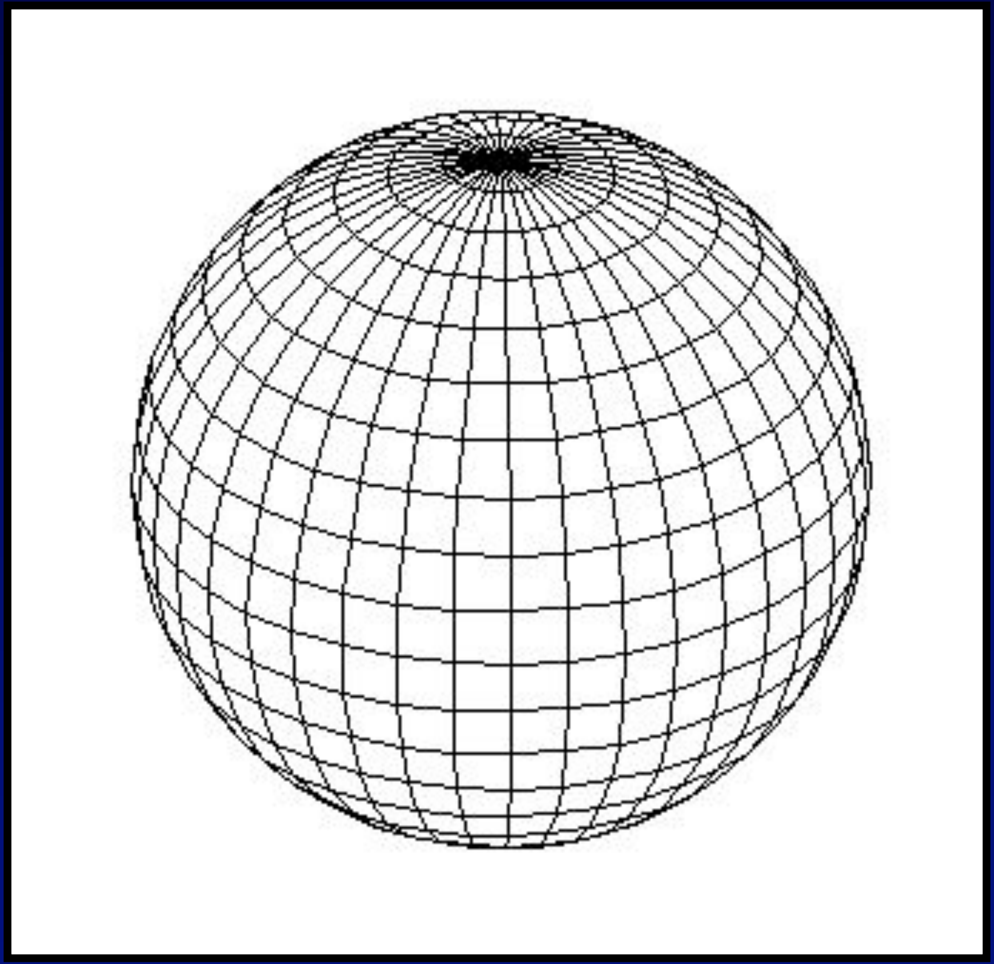
- Degree and kind of structure



Domains

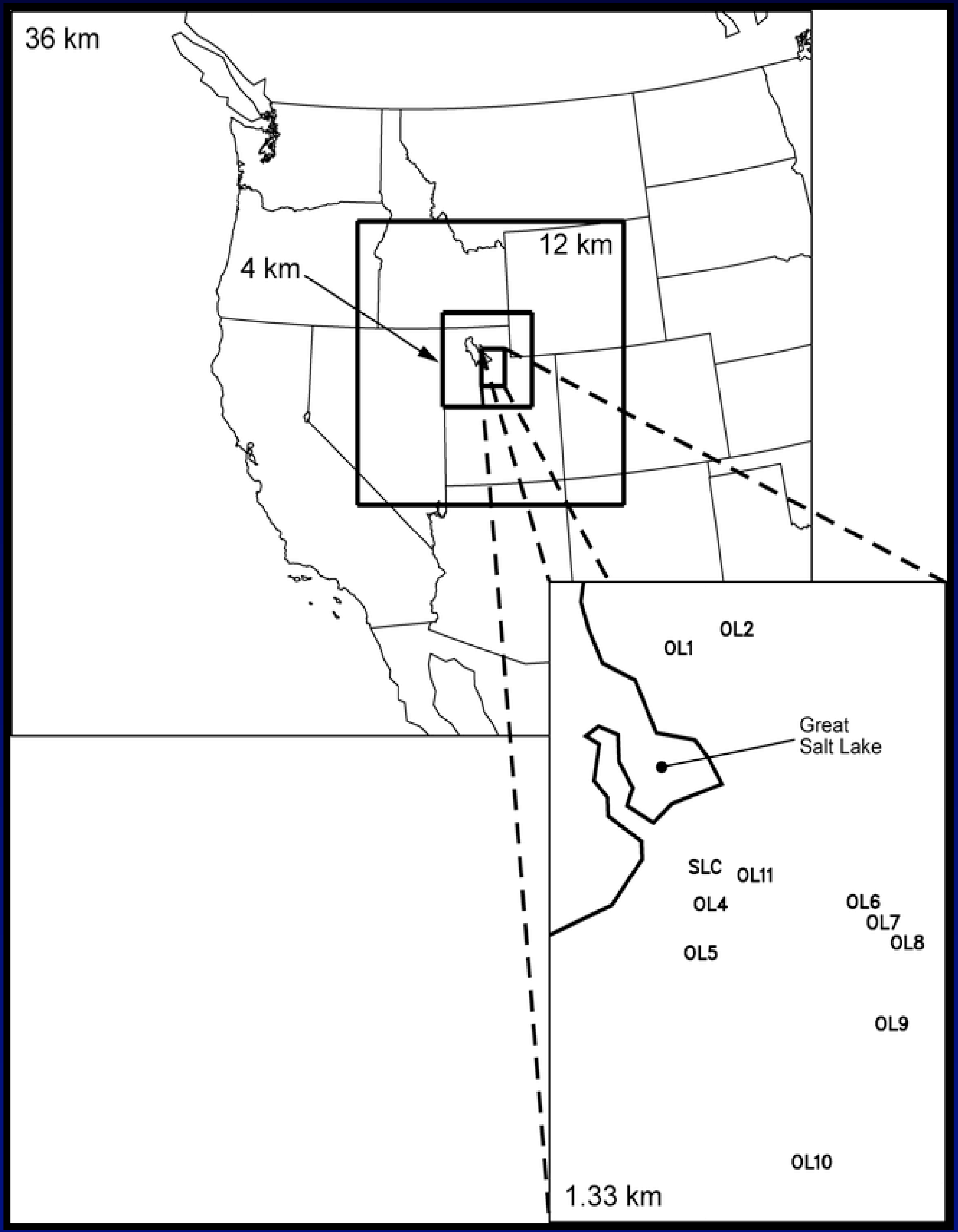
- Shape

Spherical



From mitgcm.org (2006)

Flat



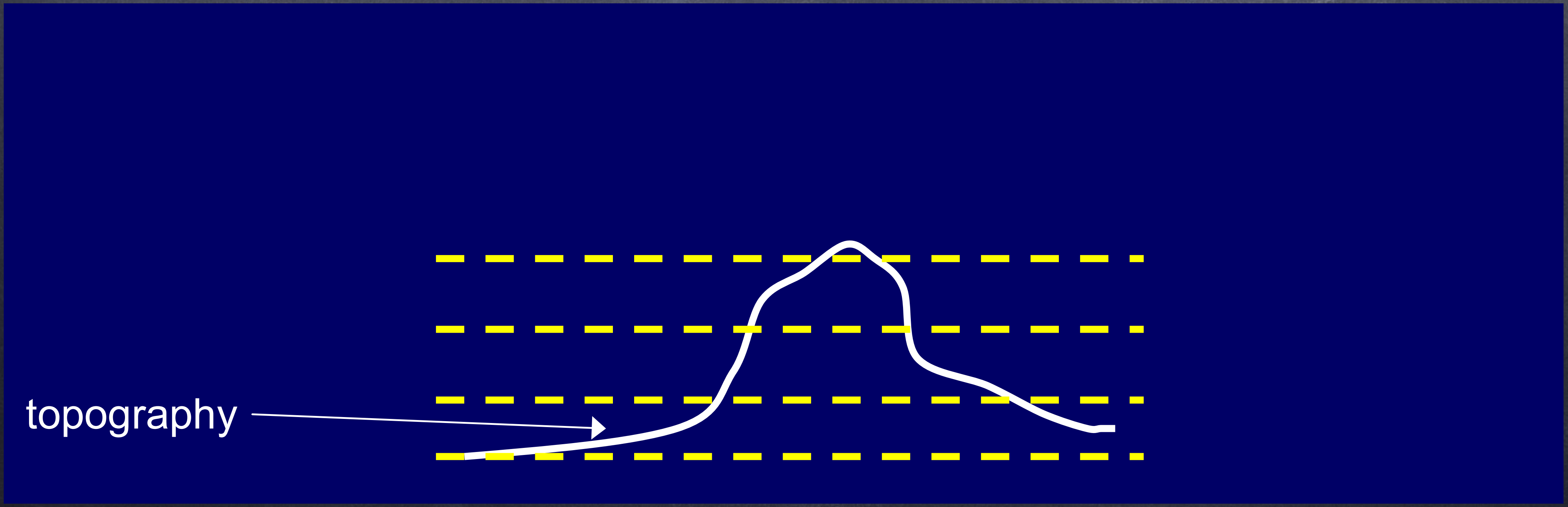
From Rife et al. (2004)

Vertical Coordinate Systems

- Height
- Pressure
- Sigma
- ETA
- Isentropic
- Hybrids

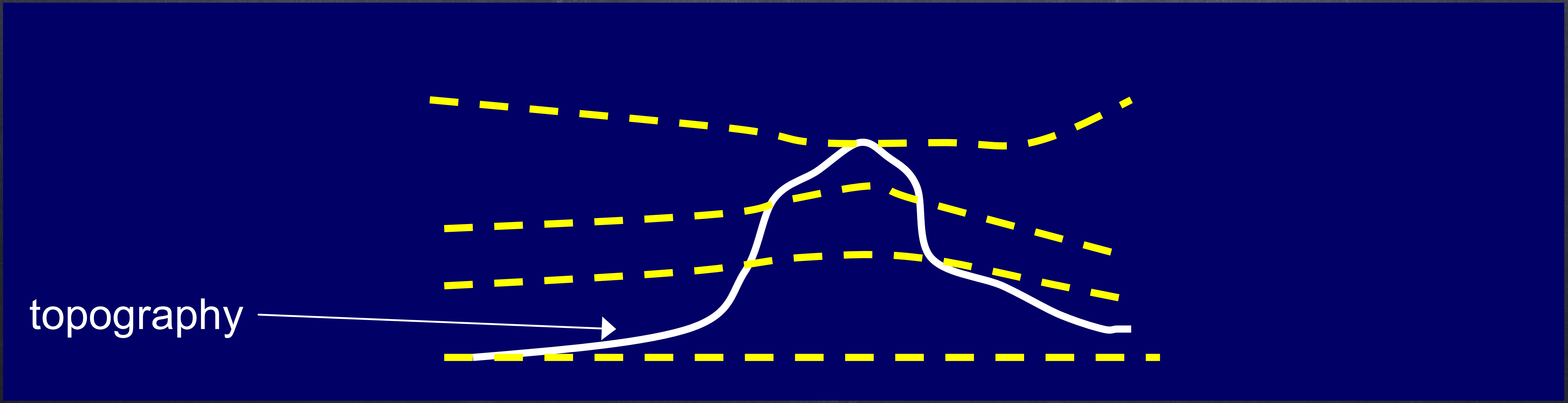
Height as a Vertical Coordinate

- Advantages
  - easy, intuitive
- Disadvantages
  - topography hard to deal with…



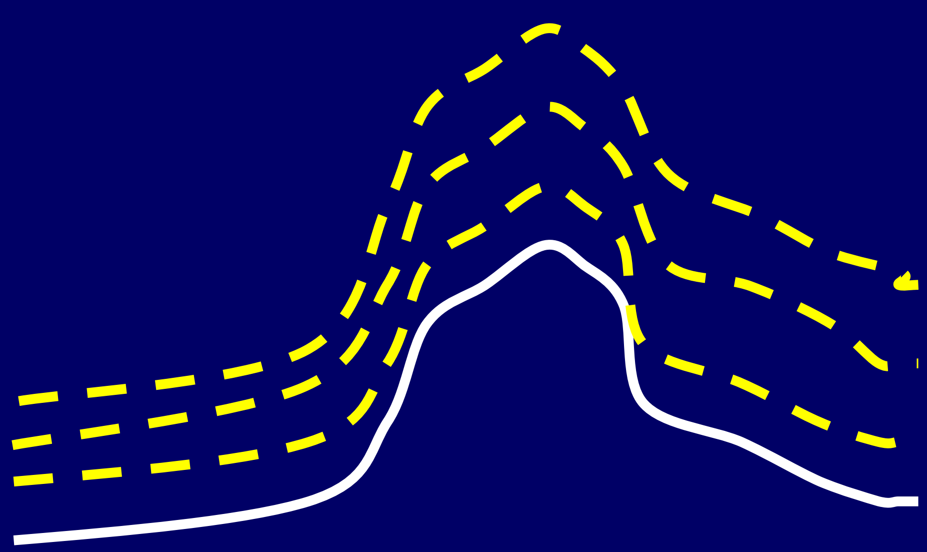
Pressure as a Vertical Coordinate

- Advantages
  - top of atmosphere is easy (p=0)
  - observations often in terms of pressure (rawinsonde, satellite)
- Disadvantages
  - pressure has same problems as height.



Sigma as a Vertical Coordinate

- Advantages: easy to represent top and bottom of atmosphere
- Disadvantages: equations need to be transformed, errors in horizontal PGF when terrain slope is steep



$$\sigma = \frac{p}{p_{sfc}}$$

•Terrain following vertical coordinate.
•Sigma = Pressure/Surface Pressure
•σ = 0 at the top of the atmosphere.
•σ = 1 at the Earth's surface.

Domains

- Vertical coordinate

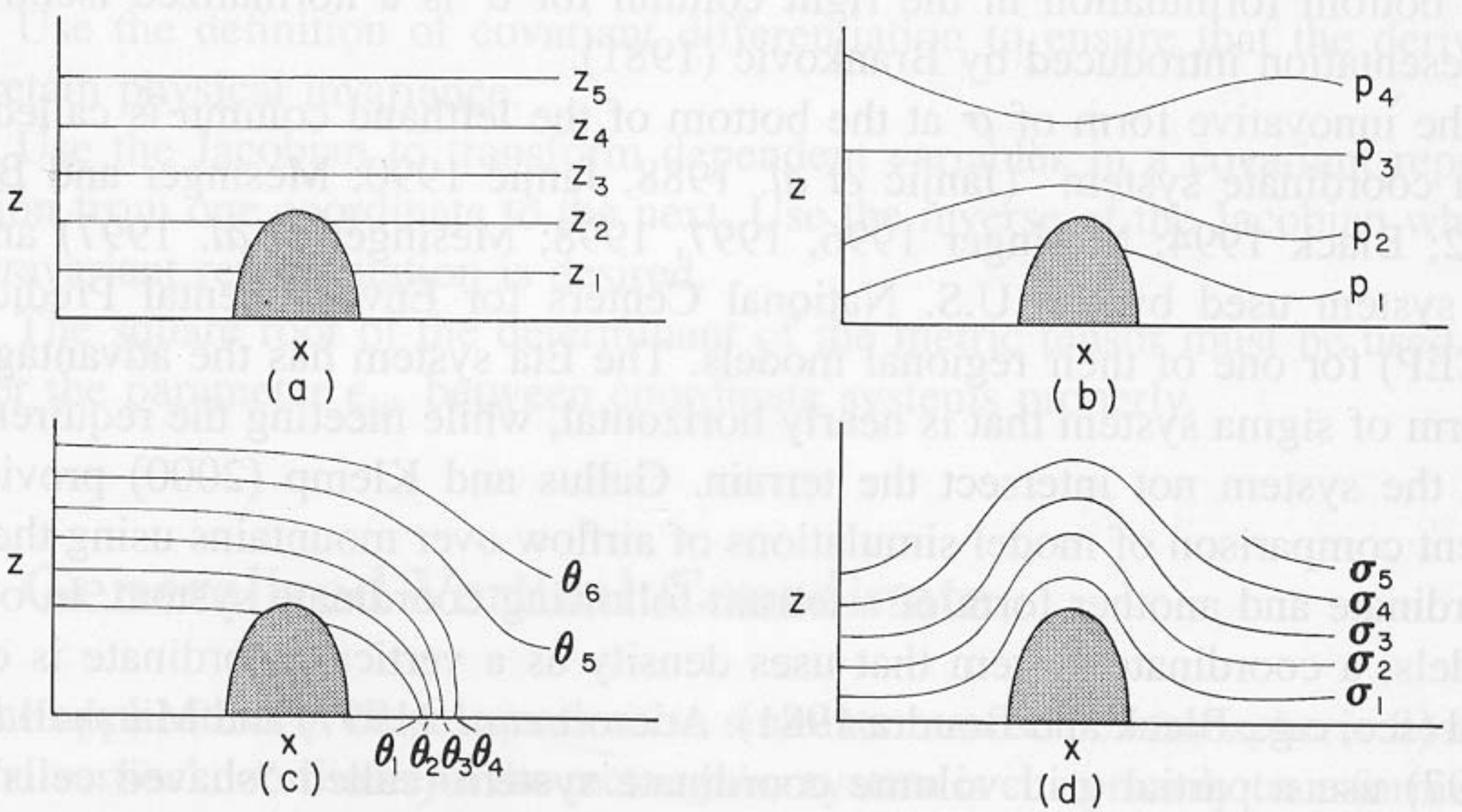


Fig. 6-2. Schematic illustrations of (a) rectangular, (b) isobaric, (c) isentropic, and (d) sigma coordinate representations as viewed in a rectangular coordinate framework.

From Pielke (2002)

ETA Coordinate

Eta as a Vertical Coordinate

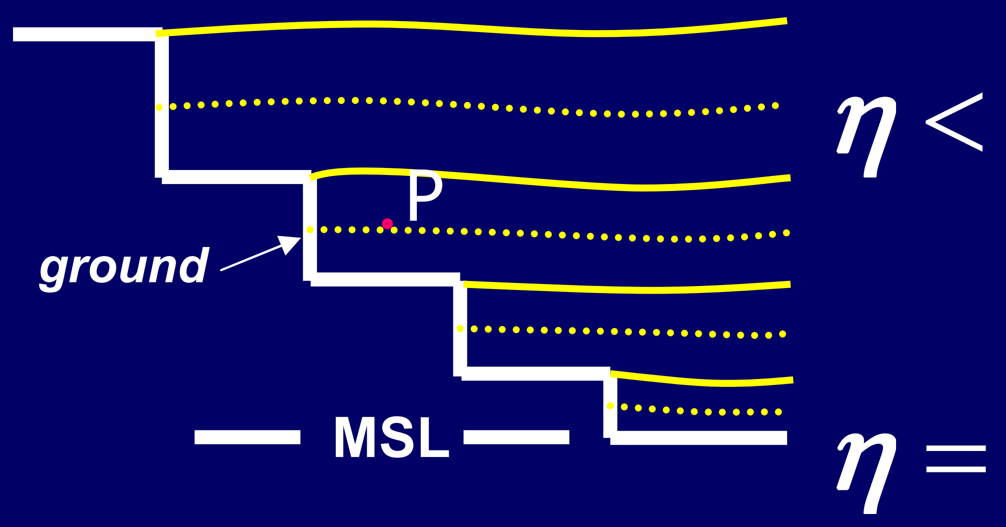
Eta is also called the stepped mountain coordinate. No holes in topography. Tries to reduce the PGF errors using sigma.

Advantage – improves calculation of horizontal pressure gradient force. Performs much better in regions of strong terrain influences

Disadvantage – does not accurately represent the surface topography.

(example NAM 218)

Hybrid pressure/sigma system



$$\eta_s = \frac{p_r(z_s) - p_t}{p_r(z=0) - p_t}$$

- pr(zs) is the pressure in the standard atmosphere at height **zs**
- pt is the pressure at the top of the atmosphere
- pr(z=0) is the pressure at sea level in the standard atmosphere

Shaved Cell Coordinate

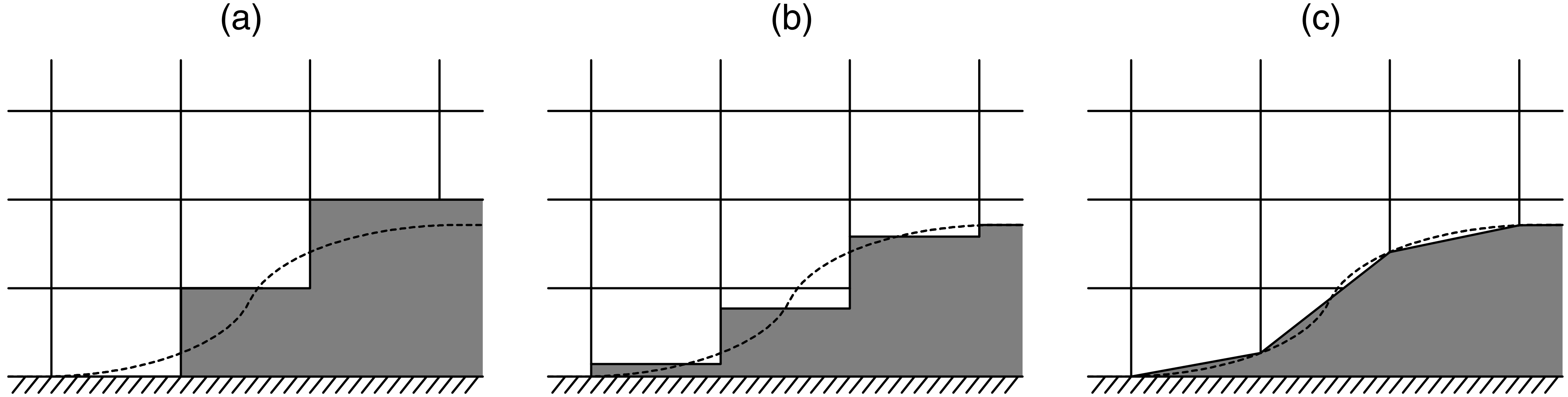


Figure 1. Three *z*-coordinate topography representations: (a) a box cell method, (b) a partial cell method, and (c) a shaved method. Solid lines and dashed lines describe the coordinates and real topography, respectively. Shaded regions describ topographic representations in each model.

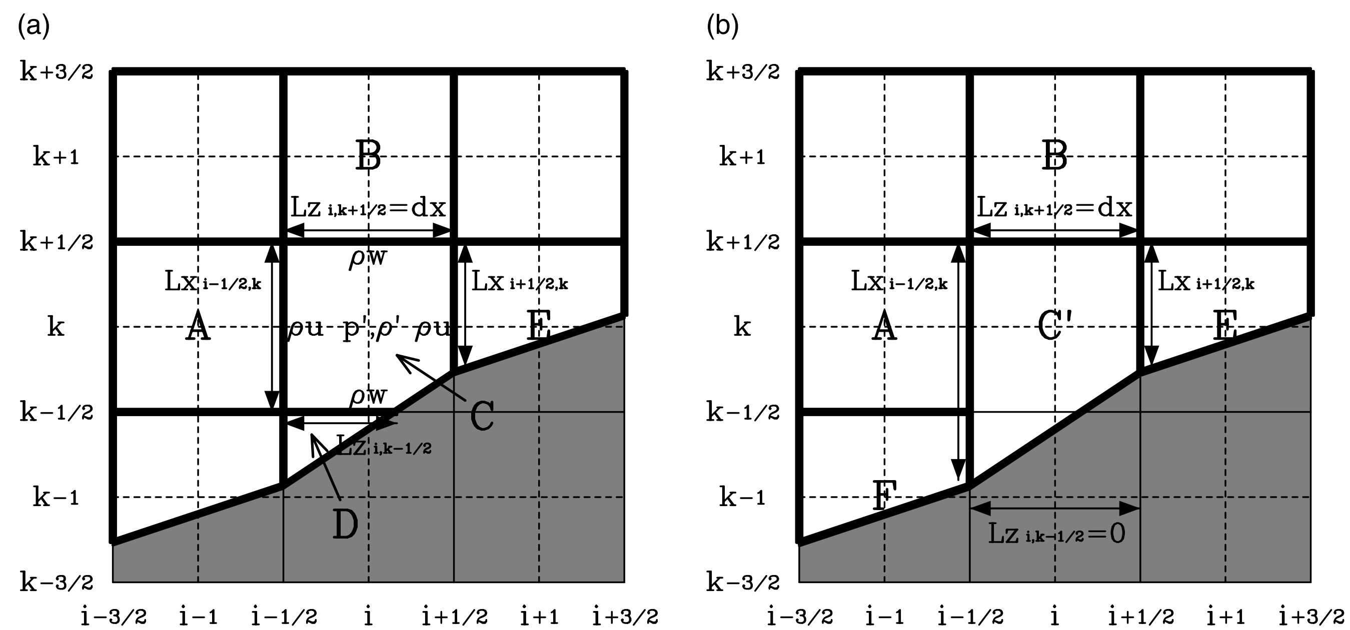
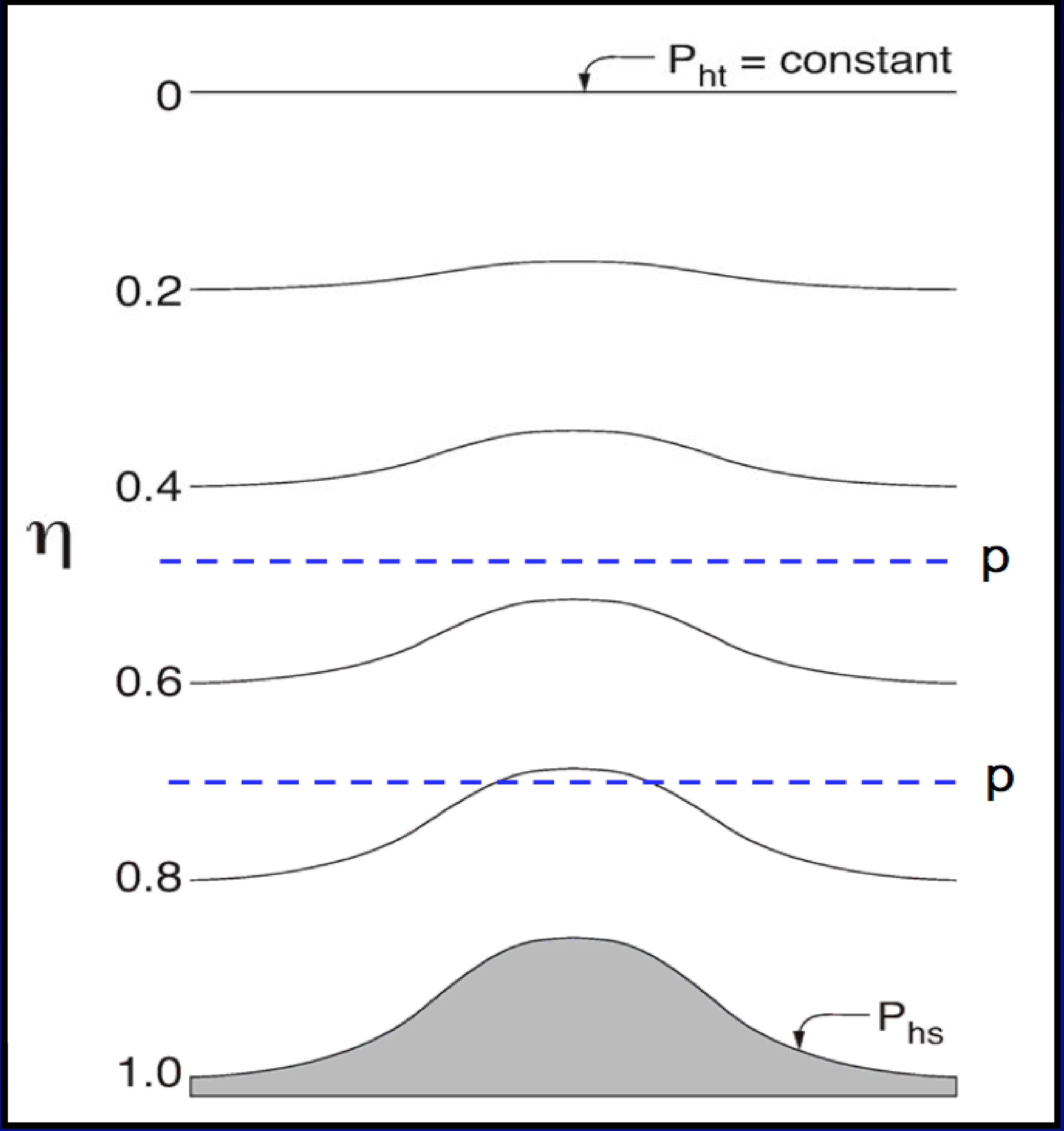


Figure 2. Combination of small cells. Thick lines describe the boundaries of the scalar cells. Shaded regions represent topography in the model. (a) Scalar cells before combination. Scalar cell C exchanges flux with the cells, A, B, D, and E. (b) Scalar cells after combining cells C and D. Combined cell C′ exchanges flux with cells A, B, E, and F.

Domains

- Vertical coordinate

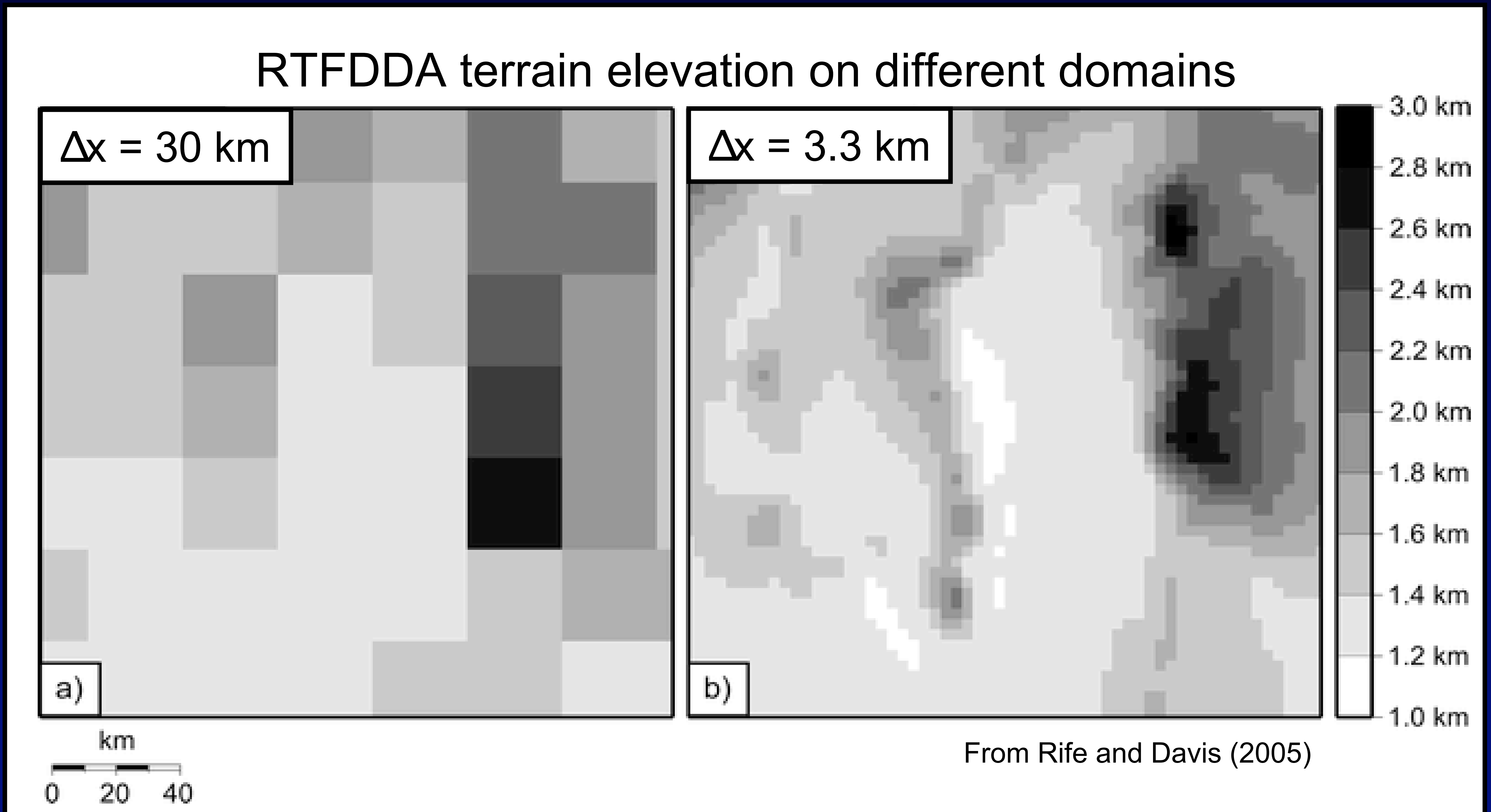
In WRF Model, vertical coordinate is normalized hydrostatic pressure, η



From Wei Wang

Domains

- Resolution



Representing PDEs

An example of from momentum equation:
U-wind accelerated by only the pressure gradient force........

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

How do you represent these on a computer?

Representing PDEs

An example of from momentum equation:
U-wind accelerated by only the pressure gradient force........

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

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How do you represent these on a computer?

Computers do arithmetic... NOT Calculus!

- Numerical methods
  - represents the continuous with discrete approximations
  - vector calculus
  - integration
  - interpolation
- Goal: convert spatial and temporal derivatives into algebraic equations that computers can solve using addition, subtraction, multiplication, and division (and a few others operations)
- Classes of numerical methods
  - Finite difference and finite volume
    - basis functions are Taylor series
  - Spectral and Galerkin methods (finite element, DG, SE)
    - based on fourier series or local polynomials

Example: Finite Differences

How to do calculus on a computer?

$$f(x \pm \Delta x) = f(x) \pm \Delta x \left.\frac{\partial f}{\partial x}\right|_x + \frac{\Delta x^2}{2!} \left.\frac{\partial^2 f}{\partial x^2}\right|_x \pm \ldots + \frac{\Delta x^n}{n!} \left.\frac{\partial^n f}{\partial x^n}\right|_x$$

Classic Taylor series expansion about “x”

To create a derivative...

$$f(x+\Delta x) - f(x-\Delta x) = 2\Delta x \left.\frac{\partial f}{\partial x}\right|_x + \frac{2\Delta x^2}{2!} \left.\frac{\partial^3 f}{\partial x^3}\right|_x + \ldots + \frac{\Delta x^{2(n+1)}}{(n+1)!} \left.\frac{\partial^{2(n+1)} f}{\partial x^{2(n+1)}}\right|_x$$

rearranging...

$$\left.\frac{\partial f}{\partial x}\right|_x = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \Delta x^2 \left.\frac{\partial^3 f}{\partial x^3}\right|_x + \ldots + \frac{\Delta x^{2n+1}}{(2n+1)!} \left.\frac{\partial^{2n+1} f}{\partial x^{2n+1}}\right|_x$$

Example: Finite Differences

- What to do with those extra derivatives?

$$\left.\frac{\partial f}{\partial x}\right|_x = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \Delta x^2 \left.\frac{\partial^3 f}{\partial x^3}\right|_x + \ldots + \frac{\Delta x^{2n+1}}{(2n+1)!}\left.\frac{\partial^{2n+1} f}{\partial x^{2n+1}}\right|_x$$

- We TRUNCATE! E.g., approximate…here to 2nd order…

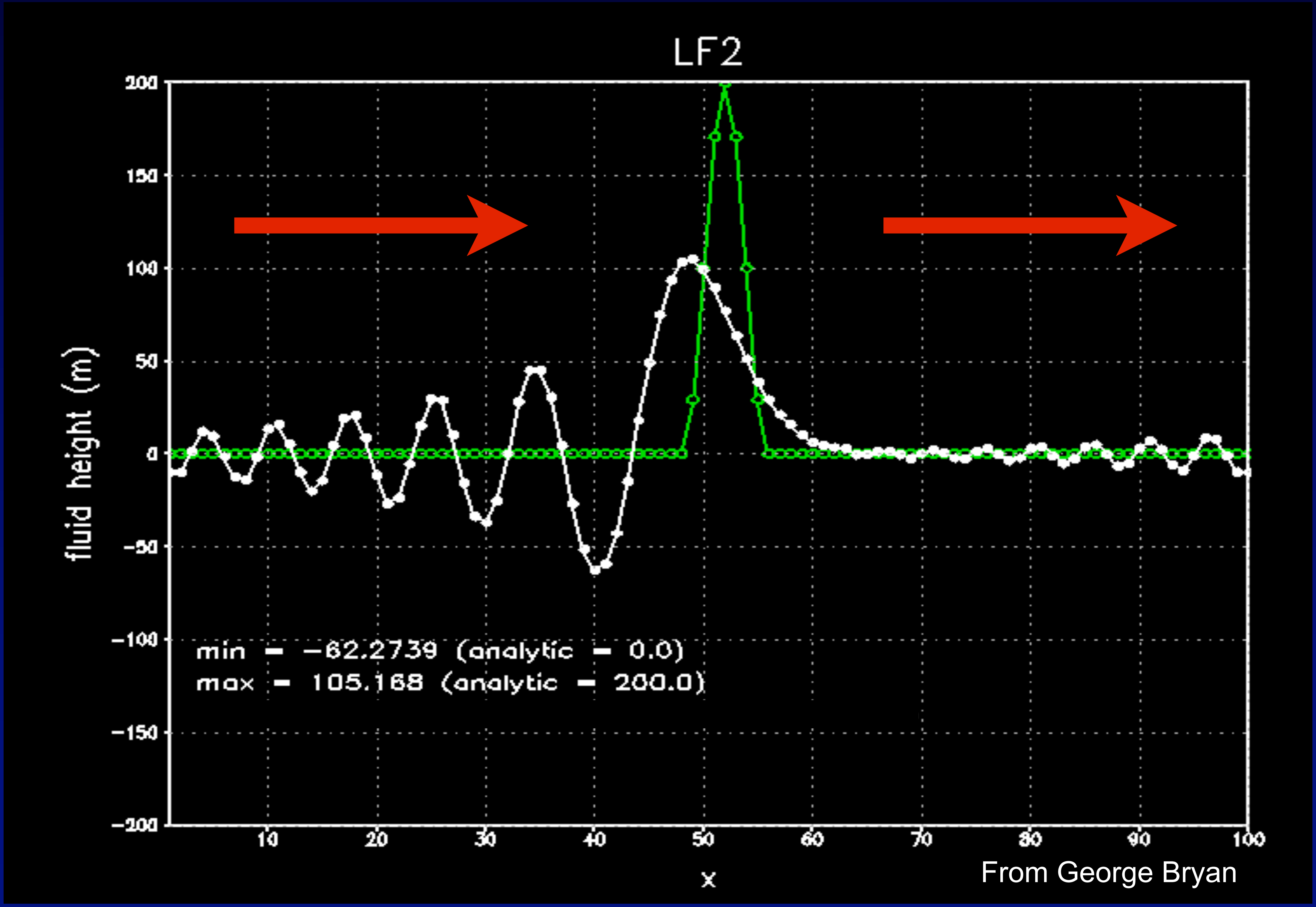
$$\left(\frac{\partial f}{\partial x}\right)_i = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \frac{f_{i-1} - f_{i+1}}{2\Delta x} + O\left(\Delta x^2\right)$$

- Truncation is always necessary (finite difference, spectral, etc).
- Truncation is one of the underlying approximation errors for the underlying PDEs
- What do these approximation errors look like in a numerical simulation?

Approximating 1D advection

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x}$$

- MM5: leapfrog (t) and 2nd-order centered (x)

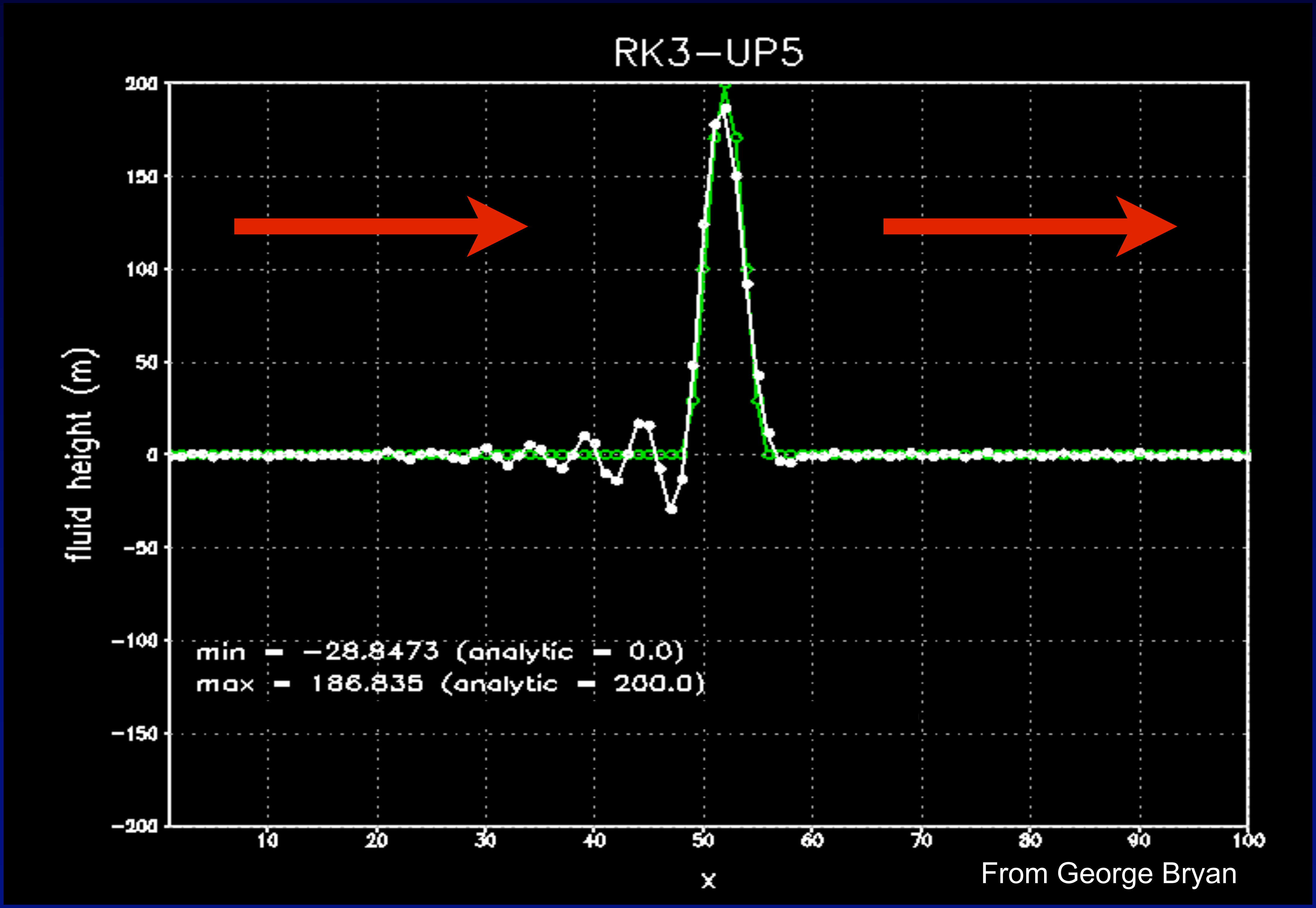


From George Bryan

Approximating 1D advection

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x}$$

- WRF: Runge-Kutta (t) and 6th-order centered (x)



From George Bryan

Summary for Approximations

- *Numerical methods do really matter!*
  - approximation errors are largest when features are smallest
  - approximations with higher-order truncation (e.g., 6th versus 2nd) have lower phase and amplitude errors for linear advection.
  - How you approximate the temporal derivatives is also important for motions....

- “Effective resolutions” for spatial finite differences approximations......
  - 2nd order FDAs: features < 16 dx are poorly represented
  - 4th order FDAs: features < 10 dx are poorly represented
  - 6th order FDAs: features < 6-7 dx are poorly represented

- Spectral models are much more accurate per “dx”, but also cost much more than finite differences. BC’s are also more complicated

- Nearly all original limited area NWP models used 2nd order approximations - despite the limits of that approximation - they still made useful predictions.
- Numerics is only part of the story - PHYSICS is also important to NWP!

What do we mean by “Physics"

- Physics: Two “categories”
  - Inputs of momentum, heat and moisture from the boundaries of the domain (earth and space)
    - friction
    - sea surface fluxes
    - solar radiation
  - processes that are too small to be resolved on a numerical grid
    - ice nucleation on CCN
    - melting of graupel into rain
    - vertical transport of heat, momentum and moisture from convective plumes in the boundary layer
- Both require PARAMETERIZATION: represent the integrated effect
- How do we formally represent this?

Physics -> Parameterizations

- Parameterizations approximate the bulk effects of physical processes too small, too brief, too complex, or too poorly understood to be explicitly represented
- In most modern models, the following parameterizations are used to represent processes to fast or small or even not well known enough….
    - cumulus convection
    - microphysical processes
    - radiation (short wave, long wave)
    - turbulence and diffusive processes
    - boundary layer and surface fluxes
    - interactions with earth’s surface (mountain drag effects)
- Many of the biggest improvements in model forecasts will come from improving these parameterizations

Reynolds Averaging

- Integrating the governing differential equations in a limited area numerically will limit the explicit representation of atmospheric motions and processes at a scale smaller than the grid interval, truncated wavelength, or finite element
- The subgrid-scale disturbances may be inappropriately represented by the grid point values, which may cause nonlinear aliasing and nonlinear numerical instability
- One way to resolve the problem is to explicitly simulate any significant small-scale motions and processes. This is called direct numerical simulation (DNS). This would require grids where Δx ~ 0.1 - 1 m.
- DNS is impractical for NWP. Models now simulate large turbulent eddies explicitly. This is called large-eddy simulations (LES).
- Reynolds averaging is the formalism which separates out the resolvable and unresolvable scales of motion in the equations themselves.
- We do so by splitting our dependent variables (u, T, q, etc.) into mean (resolved) and turbulent (perturbation/unresolved) components, e.g.,

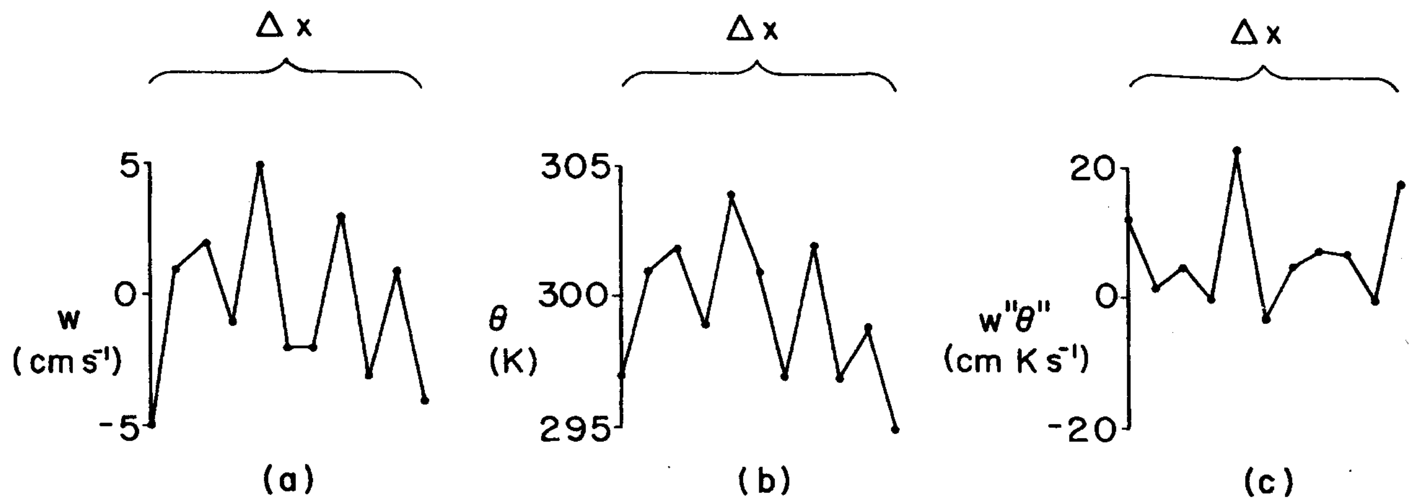
Reynolds Averaging

$$w = \overline{w} + w' \qquad \theta = \overline{\theta} + \theta'$$

$$w\theta = \overline{w}\overline{\theta} + \overline{w'\theta'} + \cancel{\overline{\overline{w}\theta'}} + \cancel{\overline{w'\overline{\theta}}}$$

In statistical terms, these fluxes, as an average of the product of deviation components, are also called *covariances*.

Figure shows the subgrid scale covariance $\overline{w'\theta'}$.



Reynolds Averaging for Bnd Layer

$$\frac{D\bar{u}}{Dt} = f\bar{v} - \frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial x} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'u'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{u'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{u'w'}\right)}{\partial z}\right] +$$

$$\frac{D\bar{v}}{Dt} = -f\bar{u} - \frac{1}{\rho_o}\frac{\partial \bar{p}}{\partial y} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'v'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{v'w'}\right)}{\partial z}\right]$$

$$\frac{D\bar{w}}{Dt} = -\frac{1}{\rho_o}\frac{\partial p_1}{\partial z} - g\frac{\rho_1}{\rho_o} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'w'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'w'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'w'}\right)}{\partial z}\right]$$

$$\frac{D\bar{\theta}}{Dt} = \overline{S_\theta} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'\theta'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'\theta'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'\theta'}\right)}{\partial z}\right] + \kappa\nabla^2\bar{\theta},$$

$$\frac{D\bar{\phi}}{Dt} = \overline{S_\phi} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'\phi'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'\phi'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'\phi'}\right)}{\partial z}\right] + \kappa\nabla^2\bar{\phi},$$

$$\phi = q_v, q_c, q_i, q_r, q_s, q_g,$$

In the above, $\overline{v'\theta'}$, and $\overline{w'\theta'}$ are turbulent heat fluxes, $\overline{u'w'}$ and $\overline{v'w'}$ are vertical turbulent fluxes of zonal momentum, and $\overline{u'v'}$ is the horizontal turbulent flux of zonal momentum.

In order to "close" the system (closure problem), the flux terms need to be represented (parameterized) by the grid-volume averaged terms (terms with "upper bar"s).

Boundary layer approximation (horizontal scales >> vertical scales), e.g. :

$$\frac{\partial \overline{u'u'}}{\partial x} \ll \frac{\partial \overline{u'w'}}{\partial z}$$

High Reynolds number approximation (molecular diffusion << turbulent transports), e.g.:

$$\nu\nabla^2 U \ll \frac{\partial \overline{u'w'}}{\partial z}$$

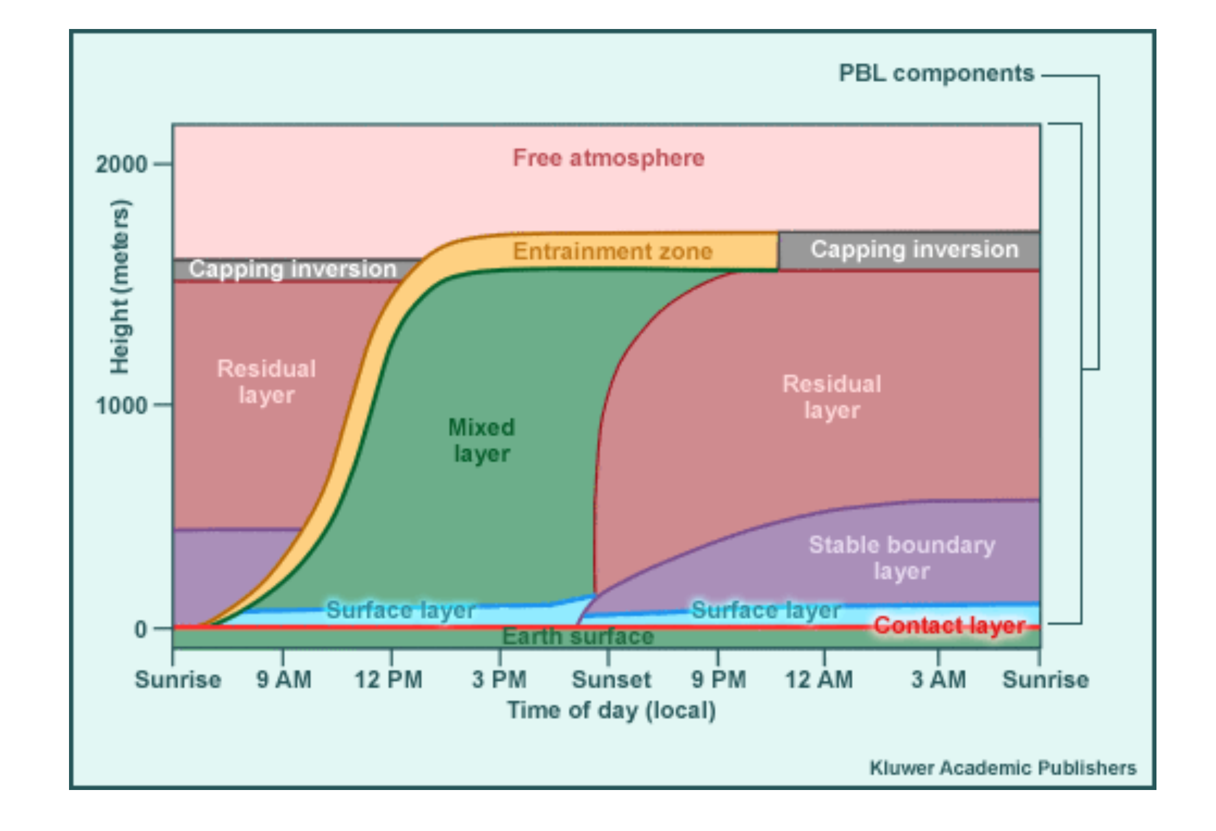
$$\boxed{\begin{aligned}\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} - fV &= -\frac{1}{\rho_o}\frac{\partial P}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z} \\ \frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + W\frac{\partial V}{\partial z} + fU &= -\frac{1}{\rho_o}\frac{\partial P}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z}\end{aligned}}$$

Reynolds Stress

Closure Problem

- Estimating those Reynolds stress terms is called the closure problem
  - to close the system of equations to be solved we need to decide how to formulate those fluxes IN TERM OF THE MEAN VARIABLES!
- Various levels of “closure”
  - 1st order (diagnostic closures)
  - 2nd order (prognostic closures)
  - 3rd and higher (here be dragons….)
- *For all closures, you end up with “picking” some coefficients or choosing an approach which approximates some process (often poorly)*

Here comes complexity!



- Planetary Boundary Layer
  - contact layer
  - surface layer
  - boundary layer
- Reynolds fluxes must account for….
  - nocturnal effect
  - stable BL boundary layer
  - neutral BL
  - convective BL
  - capping inversion
  - residual layers
  - ?????

Closure Methods

Bulk Aerodynamic Parameterization

The boundary layer is treated as a single slab and assume the wind speed and potential temperature are independent of height, and the turbulence is horizontally homogeneous.

$$\overline{u'w'} = -C_d \overline{V}^{\,2} \cos\mu\,; \quad \overline{v'w'} = -C_d \overline{V}^{\,2} \sin\mu\,; \quad \overline{w'\theta'} = -C_h \overline{V}^{\,2}\left[\overline{\theta} - \overline{\theta}_{z_o}\right],$$

Cd, Ch now need to be specified!

where C_d and C_h are nondimensional *drag and heat transfer coefficients*, respectively,

K-theory parameterization

In this approach, the turbulent flux terms in (14.1.3)-(14.1.7) are written as,

$$\overline{u'w'} = -K_m \frac{\partial \overline{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \overline{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \overline{q}}{\partial z}. \quad (14.2.1)$$

Km, Kh now need to be specified!

If the gradient terms of (14.2.1) (e.g., $\partial \overline{u} / \partial z$) are calculated based on local gradients, it is call local closure; otherwise it is called non-local closure. Normally, a non-local closure would do a better job for a convective boundary layer.

$$K_m \sim c_m L^2 \left| \frac{\partial \vec{V}}{\partial z} \right|$$

$$K_m \sim c_m L^2 \left(\frac{R_i^c - R_i}{R_i} \right) \left| \frac{\partial V}{\partial z} \right|$$

Turbulent kinetic energy (TKE or 1 1/2) closure scheme

The TKE, $(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$, is predicted, while the other subgrid scale turbulent flux terms are diagnosed and related to the TKE and to the grid-scale mean values.

$$\frac{\partial \overline{e}}{\partial t} = -\overline{\boldsymbol{V}} \cdot \nabla \overline{e} - \overline{\boldsymbol{V}' \cdot \nabla e} - (1/\rho_o)\left[(\overline{u'p'})_x + (\overline{v'p'})_y + (\overline{w'p'})_z\right] - (g/\rho_o)\overline{\rho' w'}$$

1 2 3 4

$$-\left[\left(\overline{u'u'}\,\overline{u}_x + \overline{u'v'}\,\overline{u}_y + \overline{u'w'}\,\overline{u}_z\right) + \left(\overline{u'v'}\,\overline{v}_x + \overline{v'v'}\,\overline{v}_y + \overline{v'w'}\,\overline{v}_z\right)\right. \quad (14.2.31)$$

5

$$\left. + \left(\overline{u'w'}\,\overline{w}_x + \overline{v'w'}\,\overline{w}_y + \overline{w'w'}\,\overline{w}_y\right)\right] + \nu \nabla^2 \overline{e} - \nu\left(\overline{u'^2_x} + \overline{v'^2_y} + \overline{w'^2_z}\right)$$

6 7

$$K_m \sim c_m L \sqrt{\overline{e}}$$

TKE Closure

local TKE: $E' \equiv 1/2(u'^2 + v'^2 + w'^2)$

mean TKE: $E \equiv 1/2(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

Derive equation for E by combining equations of total velocity components and mean velocity components:

$$\underbrace{\frac{\partial E}{\partial t}}_{\text{Storage}} + \underbrace{U\frac{\partial E}{\partial x} + V\frac{\partial E}{\partial y} + W\frac{\partial E}{\partial z}}_{\text{Mean flow TKE advection}} = \underbrace{-\frac{\partial}{\partial z}\overline{E'w'}}_{\text{Turbulent transport}} \underbrace{-\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z}}_{\text{Shear production}} \underbrace{-\frac{g}{\rho_o}\overline{\rho'w'}}_{\text{Buoyancy}} + \underbrace{\frac{\partial}{\partial z}\frac{\overline{p'w'}}{\rho}}_{\text{Pressure correlation}} \underbrace{-\varepsilon}_{\text{Dissipation}}$$

You still have to close buoyancy (include effects of moisture), pressure and TKE dissipation terms!

Parameterization of Moist Processes

In most mesoscale and NWP models, the majority of clouds, especially convective clouds, cannot be resolved by grid mesh and the moist variables need to be parameterized by the grid-volume mean variables.

Although in cloud models, the resolution is fine enough to roughly represent the clouds, the microphysical processes still need to be parameterized or properly represented.

The treatments of moist processes in a mesoscale model into two categories: (1) *parameterization of microphysical processes*, and (2) *cumulus parameterization*.

For parameterization of microphysical processes, two approaches have been taken: (a) explicit representation, and (b) bulk parameterization (normally referred to grid explicit microphysics, which is different from (a)).

Cumulus Parameterization

The collective effects of cumulus clouds at subgrid scale, such as the convective condensation and transport of heat, moisture, and momentum, on the larger scale environment are essential and need to be represented by grid-scale variables.

On the other hand, the large-scale forcing tends to modulate the cumulus convection, which in turn determines the total rainfall rate.

The representation of these processes is carried out by the *cumulus parameterization schemes*.

To parameterize the interaction between cumulus clouds and their environment, we must determine the relationship between cumulus convection and its larger-scale environment.

Cumulus parameterization schemes may be divided into schemes for large-scale models ($\Delta x > 50km; \Delta t > O(\mathrm{min})$) and schemes for mesoscale models ($10km < \Delta x < 50km; \Delta t < O(\mathrm{min})$).

For models having grid spacing less than 10 km, microphysics parameterization schemes are more appropriate and often employed.

Explicit Microphysics

In the bulk parameterization approach, each category of the water substance is governed by its own continuity equation.

The shape and size distributions are assumed a priori and the basic microphysical processes are parameterized.

The water substance may be divided into six categories: (1) water vapor, (2) cloud water, (3) cloud ice, (4) rain, (5) snow, and (6) graupel/hail (Orville 1980; Lin, Farley, and Orville 1983 - LFO scheme or Lin et al. scheme).

Some basic microphysical processes:

Accretion: Any larger precipitation particle overtakes and captures a smaller one.

Coalescence: The capture of small cloud droplets by larger cloud droplets or raindrops.

Autoconversion: The initial stage of the collision–coalescence process whereby cloud droplets collide and coalesce to form drizzle drops.

Aggregation: The clumping together of ice crystals to form snowflakes.

Riming: Droplets freeze immediately on contact of ice crystal will form **rimed crystal** or **graupel**. If freezing is not immediate, it may form **hail**.

The *size distributions* of rain (q_r), snow (q_s), and graupel or hail (q_g) are hypothesized as

$$N_k(D) = N_{ok}\exp(-\lambda_k D_k)\,, \qquad (14.3.6)$$

where $k = r, s, or\ g$, N_{ok} is based on observations, D_k is the diameter of the water substance, and λ_p is the *slope parameter* of the size distribution.

This type of distribution is called the *Marshall-Palmer distribution* (Marshall and Palmer 1948).

The slope parameters are given by

$$\lambda_k = \left(\frac{\pi\rho_k N_{ok}}{\rho q_k}\right)^{0.25},$$

where ρ_k is the density of water, snow or graupel.

In general, the *size distribution* (14.3.6) includes the shape factor and is written as

$$N_k(D) = N_{ok} D_k^{\alpha}\exp(-\lambda_k D_k)\,,\ k = r,\ s,\ or\ g, \qquad (14.3.10)$$

where α is called the *shape parameter*. Thus, there are 3 parameters or moments, N_{ok}, $\lambda_{k,}$ α, to be determined.

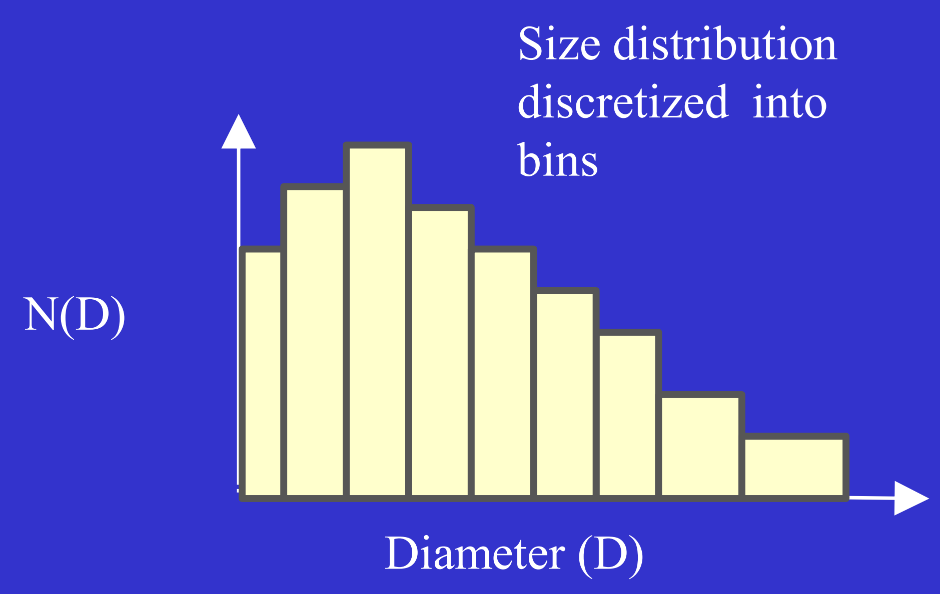
Microphysical Schemes

- Various levels of complexity
- Single moment
  - predict mixing ratio (lambda)
  - Fix N_0, alpha (impacts reflectivity factor Z)
- Double moment
  - predict mixing ratio, N_0
  - alpha is fixed
- "2.5" scheme: diagnose alpha from mean variables and type of particle
- 3 moment - predict q, N_0 and Z.
- Bin models
  - break distribution into "bins" (like 100-200 bins)
  - prediction of interactions between all bins
  - just now feasible for water and ice in 3D cloud models (Ted Manselll)

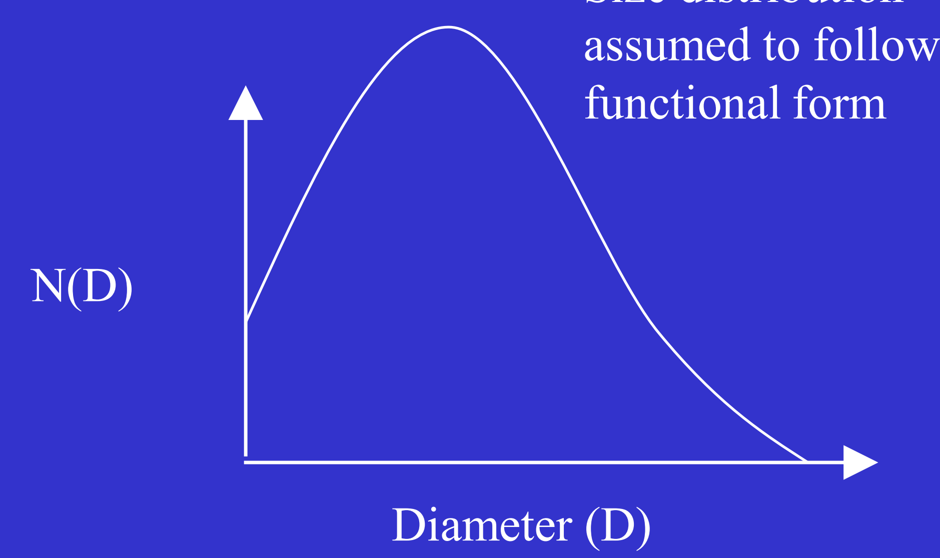
Examples

Microphysics schemes can be broadly categorized into two types:

Detailed (bin)



bulk



Representation of particle size distribution

1 Mom. Microphysical Parameterizations

The microphysical processes are very complicated, which are summarized in Fig. 14.6. (From Lin et al. 1983 – the Lin-Farley-Orville Scheme; MM5 Goddard scheme and several other schemes are based on LFO scheme)

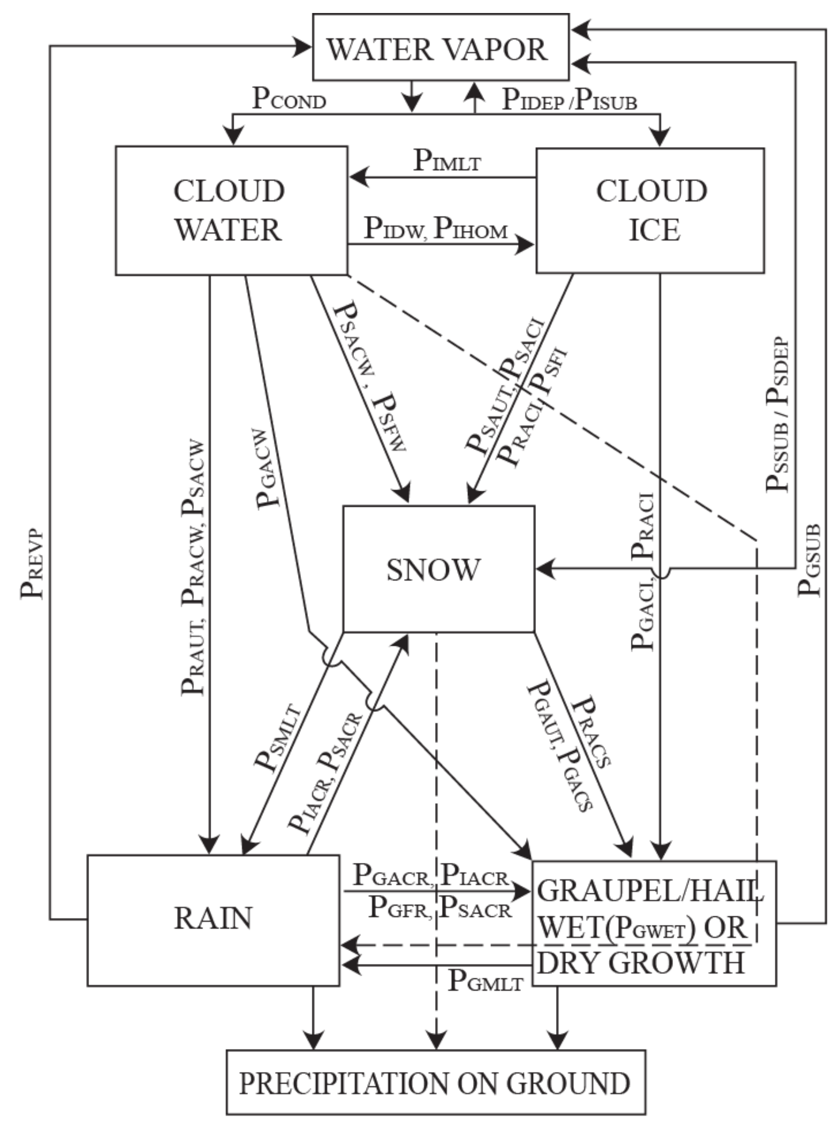


Fig. 14.6: A sketch of cloud microphysical processes in a bulk microphysics parameterization (LFO) scheme including ice phase. Meanings of the production terms (i.e., P terms) can be found in Table 14.1. (Adapted after Lin, Farley, and Orville 1983; Orville and Kopp 1977) (Lin 2007)

2 Mom. Microphysical Parameterizations

Continuity Equations

The prognostic equations for the mixing ratios of all phases of water in the parameterization (i.e., vapor, liquid, ice, and liquid water on ice) are as follows:

$$\frac{dq_v}{dt} = -\mathrm{QCND} - \mathrm{QREVP} - (1-\delta)(\mathrm{QSEVP} + \mathrm{QGEVP} + \mathrm{QHEVP}) - \delta(\mathrm{QINT} + \mathrm{QIDEP} + \mathrm{QSDEP} + \mathrm{QGDEP} + \mathrm{QHDEP}), \quad (\mathrm{A.1})$$

$$\frac{dq_w}{dt} = \mathrm{QCND} - \mathrm{QRAUT} - \mathrm{QRACW} - \mathrm{QSACWS} - \mathrm{QGACWG} - \mathrm{QHACWH} - \mathrm{QIFM} - \delta(\mathrm{QIACW} + \mathrm{QIHR} + \mathrm{QSACWG} + \mathrm{QGACWH} + \mathrm{QHACWG}), \quad (\mathrm{A.2})$$

$$\frac{dq_i}{dt} = \mathrm{QIFM} + \delta(\mathrm{QINT} + \mathrm{QIDEP} + \mathrm{QIACW} + \mathrm{QIHR} + \mathrm{QIHMS} + \mathrm{QIHMG} + \mathrm{QIHMH} - \mathrm{QICNVS} - \mathrm{QRACI} - \mathrm{QSACI} - \mathrm{QGACI} - \mathrm{QHACI}), \quad (\mathrm{A.3})$$

$$\frac{dq_r}{dt} = \mathrm{QREVP} + \mathrm{QRAUT} + \mathrm{QRACW} + \mathrm{QSSHD} + \mathrm{QGSHD} + \mathrm{QHSHD} - \delta(\mathrm{QIACR} + \mathrm{QSACRS} + \mathrm{QSACRG} + \mathrm{QSACRH} + \mathrm{QGACRG} + \mathrm{QGACRH} + \mathrm{QHACR}), \quad (\mathrm{A.4})$$

$$\frac{dq_s}{dt} = \mathrm{QSACWS} - \mathrm{QGACS} - \mathrm{QHACS} - \mathrm{QSSHD} + (1-\delta)\mathrm{QSEVP} + \delta(\mathrm{QSDEP} + \mathrm{QICNVS} + \mathrm{QSACI} + \mathrm{QSACRS} - \mathrm{QRACSG} - \mathrm{QRACSH} - \mathrm{QWACSG} - \mathrm{QIHMS}), \quad (\mathrm{A.5})$$

$$\frac{dq_g}{dt} = \mathrm{QGACWG} + \mathrm{QGACS} - \mathrm{QGSHD} + (1-\delta)\mathrm{QGEVP} + \delta(\mathrm{QGDEP} + \mathrm{QGACI} + \mathrm{QGACRG} + \mathrm{QSACRG} + \mathrm{QRACSG} + \mathrm{QSACWG} + \mathrm{QWACSG} + \mathrm{QHACWG} + \mathrm{QWACHG} - \mathrm{QRACGH} - \mathrm{QWACGH} - \mathrm{QIHMG}), \quad (\mathrm{A.6})$$

$$\frac{dq_h}{dt} = \mathrm{QHACWH} + \mathrm{QHACS} - \mathrm{QHSHD} + (1-\delta)\mathrm{QHEVP} + \delta(\mathrm{QHDEP} + \mathrm{QHACI} + \mathrm{QHACR} + \mathrm{QIACR} + \mathrm{QRACI} + \mathrm{QSACRH} + \mathrm{QRACSH} + \mathrm{QGACRH} + \mathrm{QRACGH} + \mathrm{QGACWH} + \mathrm{QWACGH} - \mathrm{QWACHG} - \mathrm{QIHMH}), \quad (\mathrm{A.7})$$

$$\frac{dq_{sw}}{dt} = \mathrm{QSACW} - \mathrm{QSFM} - \mathrm{QSSHD} - F_{sw}(\mathrm{QGACS} + \mathrm{QHACS}) + (1-\delta)\mathrm{QSEVP} + \delta[\mathrm{QSACRS} - F_{sw}(\mathrm{QRACSG} + \mathrm{QRACSH} + \mathrm{QWACSG})], \quad (\mathrm{A.8})$$

$$\frac{dq_{gw}}{dt} = \mathrm{QGACW} - \mathrm{QGFM} - \mathrm{QGSHD} + F_{sw}\cdot\mathrm{QGACS} + (1-\delta)\mathrm{QGEVP} + \delta[\mathrm{QGACRG} + \mathrm{QSACRG} + \mathrm{QSACWG} + \mathrm{QHACWG} + F_{sw}(\mathrm{QRACSG} + \mathrm{QWACSG}) + F_{hw}\cdot\mathrm{QWACHG} - F_{gw}(\mathrm{QRACGH} + \mathrm{QWACGH})], \quad (\mathrm{A.9})$$

$$\frac{dq_{hw}}{dt} = \mathrm{QHACW} - \mathrm{QHFM} - \mathrm{QHSHD} + F_{sw}\cdot\mathrm{QHACS} + (1-\delta)\mathrm{QHEVP} + \delta[\mathrm{QIACR} + \mathrm{QSACRH} + \mathrm{QGACRH} + \mathrm{QHACR} + \mathrm{QGACWH} + F_{sw}\cdot\mathrm{QRACSH} + F_{gw}(\mathrm{QRACGH} + \mathrm{QWACGH}) - F_{hw}\cdot\mathrm{QWACHG}]. \quad (\mathrm{A.10})$$

The functions δ in (A.1)–(A.10) and F_{xw} in (A.8)–(A.10) are defined as

$$\delta = \begin{cases} 1, & T < 0°\mathrm{C} \\ 0, & \text{otherwise}, \end{cases} \quad (\mathrm{A.11})$$

$$F_{xw} = q_{xw}/q_x, \quad (\mathrm{A.12})$$

where the variable x represents the precipitation ice species of snow, graupel, and hail/frozen drops ($x = s, g, h$).

Changes in the simulated potential temperature (q) due to latent heating are calculated using the following thermodynamic energy equation:

$$\frac{d\theta}{dt} = \frac{L_v}{\Pi C_p}(\mathrm{QCND} + \mathrm{QREVP}) + \frac{\delta L_s}{\Pi C_p}(\mathrm{QINT} + \mathrm{QIDEP} + \mathrm{QSDEP} + \mathrm{QGDEP} + \mathrm{QHDEP}) + \frac{L_f}{\Pi C_p}[\mathrm{QIFM} + \mathrm{QSFM} + \mathrm{QGFM} + \mathrm{QHFM} + \delta(\mathrm{QIACW} + \mathrm{QIHR})], \quad (\mathrm{A.13})$$

where Π is the Exner function $(p_0/p)^\kappa$ and $\kappa = R_d/C_p$.

Finally, prognostic equations for the number concentrations of each ice species are

$$\frac{dn_i}{dt} = \mathrm{NIFM} + \delta(\mathrm{NINT} + \mathrm{NIDEP} + \mathrm{NIHMS} + \mathrm{NIHMG} + \mathrm{NIHMH} + \mathrm{NIHR} - \mathrm{NICNV} - \mathrm{NIACI} - \mathrm{NRACI} - \mathrm{NSACI} - \mathrm{NGACI} - \mathrm{NHACI}), \quad (\mathrm{A.14})$$

$$\frac{dn_s}{dt} = \mathrm{NSBR} - \mathrm{NSACS} - \mathrm{NGACS} - \mathrm{NHACS} + (1-\delta)(\mathrm{NSEVP} - \mathrm{NSSHD}) + \delta(\mathrm{NSCNV} + \mathrm{NSDEP} - \mathrm{NRACSG} - \mathrm{NRACSH} - \mathrm{NWACSG}), \quad (\mathrm{A.15})$$

$$\frac{dn_g}{dt} = (1-\delta)(\mathrm{NGEVP} - \mathrm{NGSHD}) + \delta(\mathrm{NGDEP} + \mathrm{NSACRG} + \mathrm{NWACSG} + \mathrm{NWACHG} - \mathrm{NRACGH} - \mathrm{NWACGH}), \quad (\mathrm{A.16})$$

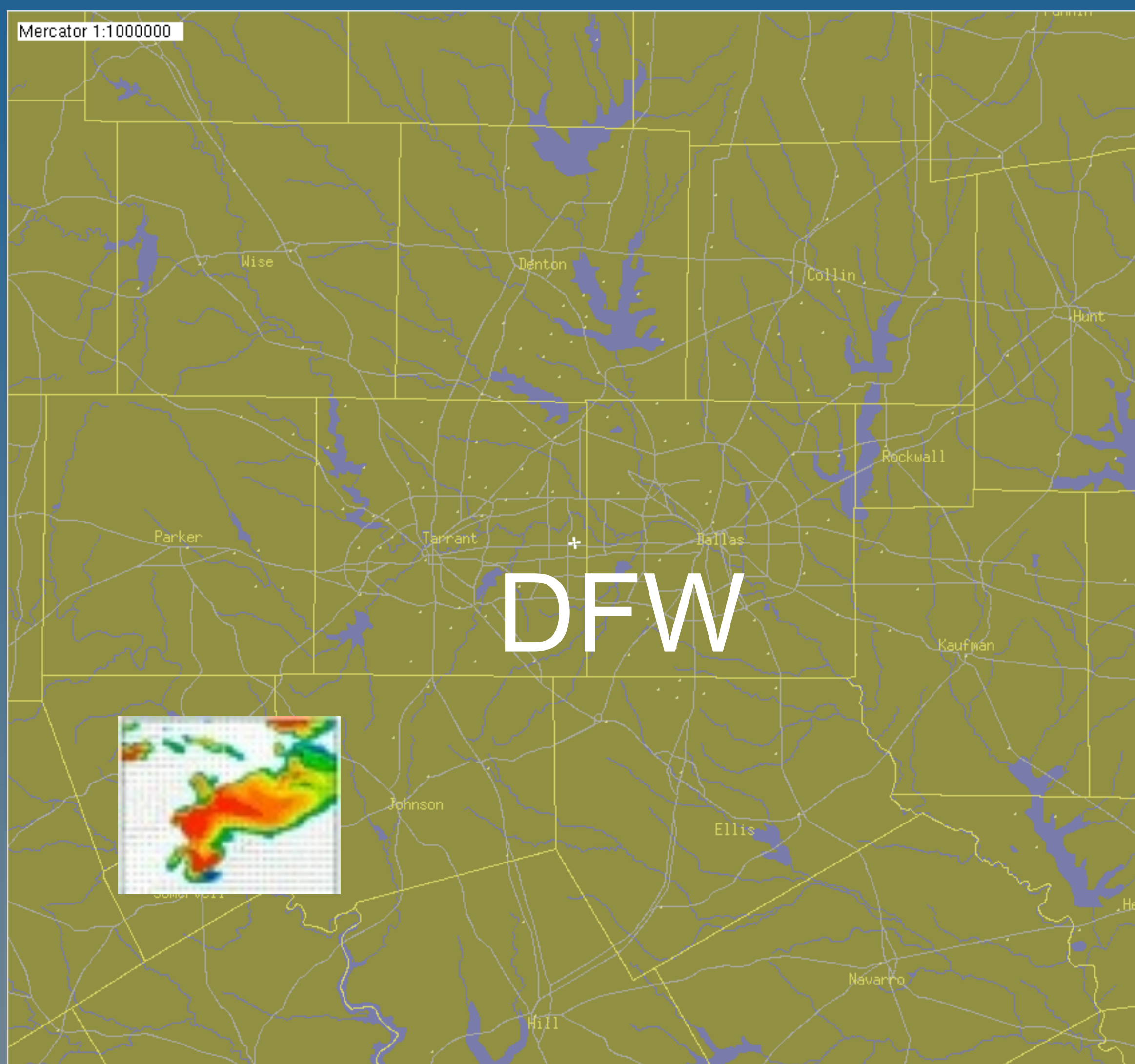
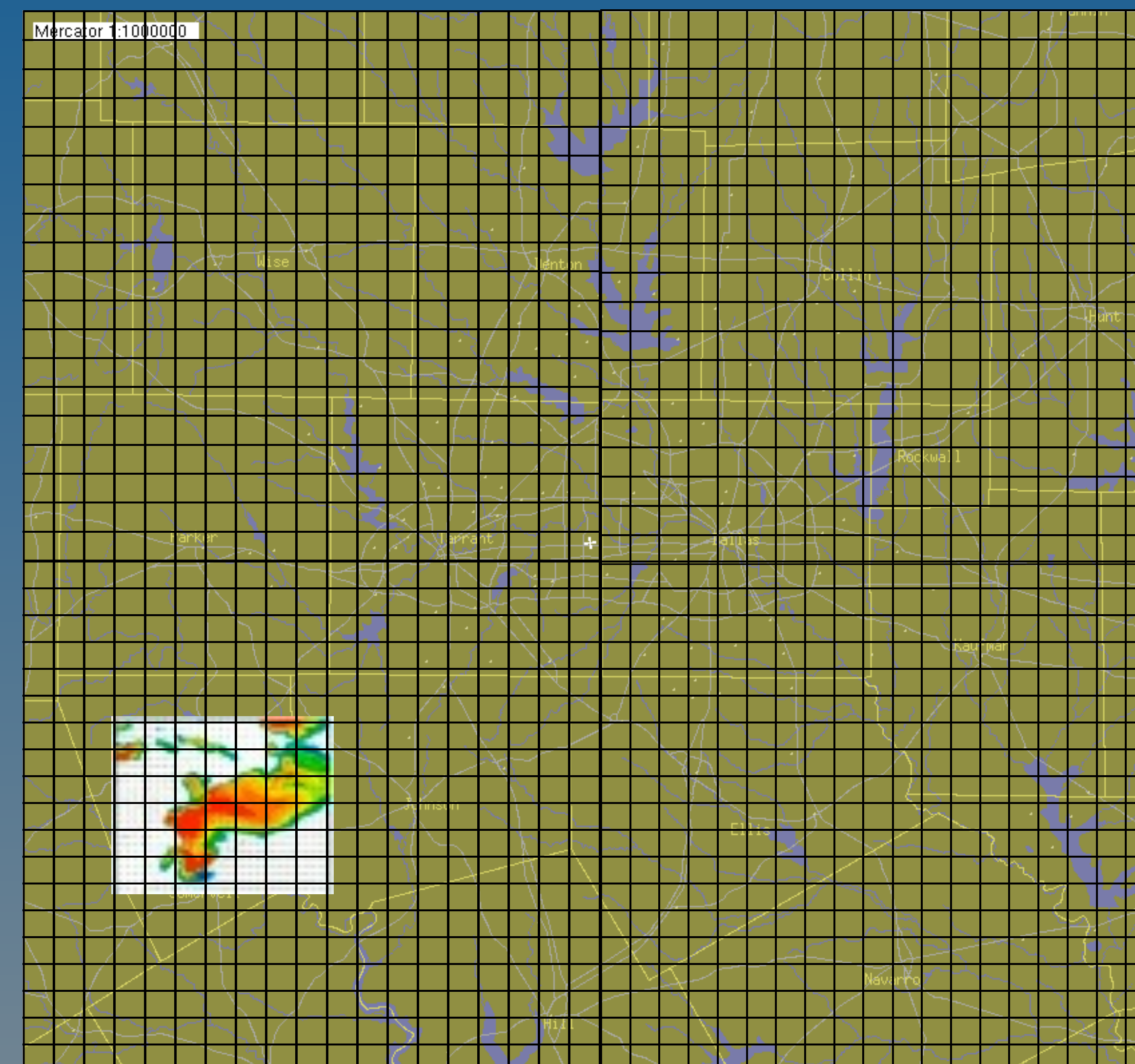
$$\frac{dn_h}{dt} = (1-\delta)(\mathrm{NHEVP} - \mathrm{NHSHD}) + \delta(\mathrm{NHDEP} + \mathrm{NIACR} + \mathrm{NSACRH} + \mathrm{NGACRH} + \mathrm{NWACGH} - \mathrm{NWACH}). \quad (\mathrm{A.17})$$

Ferrier JAS 1994

NWP in 100 min… what have ignored?

- Initial conditions
- Boundary conditions
- Various systems of equations
  - hydrostatic
  - non-hydrostatic
  - form of the equations
    - conservative
    - non-conservative
    - hamiltonian
- Parameterizations
  - radiation
  - microphysics
  - land surface
  - aerosols

How far have we come?
Resolving (sort of) a single storm!

1975	2005
	
LFM Grid Point ($\Delta x \sim 190$ km)	WRF Grid ($\Delta x \sim 4$ km)
7 vertical levels	50 vertical levels

A ~35,000x increase in CPU due to grid! (really more like ~10^6 increase with physics changes)
A typical forecast today (1 hour wallclock) would require > 5 years to run on a 1975 computer!

References/Attributions

- http://www.mmm.ucar.edu/wrf/users/workshops/WS2010/presentations/Lectures/morrison_wrf_workshop_2010_v2.pdf
- http://www.atmos.illinois.edu/~snesbitt/ATMS597R/notes/pbl.pdf
- http://www.mesolab.us/2.EES_NWP/Ch5_Lecture_Note%20(parameterizations).pdf
- Jason Knievel (NWP and WRF model)
- http://derecho.math.uwm.edu/classes/NWP/sec3-1.ppt
- Parameterization Schemes (book), D. Stensrud
- https://dl.dropboxusercontent.com/u/4017006/Mesinger_ArakawaGARP.pdf

Initial and boundary conditions

- Idealized lateral boundary conditions
  - Open
  - Rigid
  - Periodic

- Operational lateral boundary conditions
  - Generally updated during simulations
  - Not needed for global models, only for limited-area models (LAMs), such as RTFDDA
  - Can come from larger domains of same/different model or from global model
    - For RTFDDA, source is NAM (was Eta, now NMM-WRF)

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Reynolds Averaging Example

Starting with the simplest u-momentum equation,

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

By applying the Reynolds average assumptions…

$$\frac{\partial \overline{u}}{\partial t} = -\overline{u}\frac{\partial \overline{u}}{\partial x} - \overline{v}\frac{\partial \overline{u}}{\partial y} - \overline{w}\frac{\partial \overline{u}}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \overline{u'\frac{\partial u'}{\partial x}} - \overline{v'\frac{\partial u'}{\partial y}} - \overline{w'\frac{\partial u'}{\partial z}}$$

the last three RHS terms are the unresolved turbulent fluxes

$$\frac{\partial \overline{u}}{\partial t} = -\overline{u}\frac{\partial \overline{u}}{\partial x} - \overline{v}\frac{\partial \overline{u}}{\partial y} - \overline{w}\frac{\partial \overline{u}}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \boxed{\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}}$$

those fluxes can be used to account for many processes….