Chapter 2

2.8 In the oceanographic literature, the unit of mass transport is the sverdrup (Sv) (millions of cubic meters per second). The transport of ocean water by the Gulf Stream is estimated to be on the order of 150 Sv. Compare the transport by the Gulf Stream with the transport by the tradewinds estimated in the Exercise 1.17.

Solution: 1 Sv=10^6m^{3}s^{-1}. The density of water is ρ =10^3kg m^{-3}. Therefore the mass transport Tr by the Gulf Stream is

$$Tr = 150 \times 10^6 \times 10^3 \text{ kg s}^{-1} = 1.5 \times 10^{11} \text{ kg s}^{-1}.$$

Comparing this result with the transport by the trade winds (In the book's answer of Exercise 1.16, the estimate was about 1.2×10^{11} kg s^{-1}), we see that the two transports are of same order of magnitude.

2.9 If the air flowing equatorward in the tradewinds in Exercise 1.20 contains 20 g of water vapor per kg of air, estimate the mean rainfall rate within the equatorial zone (15°N–15°S) attributable to this transport.

Solution: The volume of water entering the region is given by the air transport ($Tr = 1.18 \times 10^{11}$ kg s^{-1}) times the mass of water per unit mass of air ($q = 20$ g kg^{-1}) divided by the density of water ($\rho = 10^3$ kg m^{-3}). Assuming that all the water entering becomes rain, the mean rainfall rate R over the area A is

$$R = \frac{q \times Tr}{\rho \times A}$$

If we treat the surface between 15°N and 15°S as a cylinder, the area A is:

$$\begin{aligned} A &= 2 \times \pi \times R_E \times d, \quad \text{where } d = 2 \times R_E \times \sin(15^\circ); \\ A &= 4 \times \pi \times R_E^2 \sin(15^\circ). \end{aligned}$$

Therefore

$$\begin{aligned} R &= \frac{20 \times 10^{-3} \times 1.18 \times 10^{11}}{10^3 \times 4 \times \pi \times (6.37)^2 \times 10^{12} \times \sin(15)} \\ &= \frac{20 \times 1.18}{4 \times \pi \times (6.37)^2 \times \sin(15)} \times 10^{-7} \\ &= 0.2 \times 10^{-7} \text{m s}^{-1} = 1.7 \text{ mm day}^{-1}. \end{aligned}$$

2.10 By how much would global sea level rise if both the Greenland and Icelandic ice sheets were to entirely melt?

Solution: Based on the information presented in Table 2.2, the combuned mass of the Greenland and Antarctic ice sheets is $\frac{58}{5} = 11.6$ times as great as the mass of the Greenland ice sheet alone. In Exercise 2,2 it was shown that the melting of the Greenland ice sheet would result in a 7 m rise in global sea level. Extrapolating ftom that result it can be inferred that the metting of both the Greenland and antarctic ice sheets would cayse global sea level to rise by $11.6 \times 7 = 79$ m.

2.12 (a) Describe how the level of the lake in the previous exercise would vary in response to time variations in precipitation of the form

$$P = P_0 + P' \cos(2\pi t/T)$$

where $P_0 = E_0$ does not vary with time. Show that the amplitude of the response is directly proportional to the period of the forcing. Does this help explain the prevalence of decade-to-decade variability in Fig. 2.22, as opposed to year-to-year variability? (b) Taking into account the finite depth of the lake, describe qualitatively the character of the response if P_0 were to gradually (i.e., on a time scale much longer than T) drop below the equilibrium value $P_0 = E_0$.

Solution:

2.13 Consider a lake that drains an enclosed basin as in Exercise 2.11, but in this case assume that the lake bottom is shaped like an inverted cone or pyramid. (a) Show that

$$\frac{dz}{dt} = \frac{PA}{z^2} - E_0 a_1$$

where a_1 is the area of the lake when it is 1 m deep, expressed in dimensionless units. [**Hint**: Note that the area of the lake is $a_1 z^2$ and the volume is $\frac{1}{3} a_1 z^3$.] (b) If precipitation falls over the basin at the steady rate P_0, show that the equilibrium lake level is

$$z_0 = \sqrt{\frac{P_0 A}{E_0 a_1}}$$

(c) Describe in physical terms how and why the response to a time varying forcing is different from that in Exercise 2.11. Why do residents of Salt Lake City and Astrakhan have reason to be grateful for this difference?

2.14 Reconcile the mass of oxygen in the atmosphere in Table 2.4 with the volume concentration given in Table 1.1.

Solution: In Table 2.4 the mass of oxygen per unit area is given as $m_{O_2} = 2.353 \times 10^3$ kg m^{-2}. Using the relation

$$m_{O_2} = \frac{c_{O_2} \times M_{O_2}}{\sum c_i M_i} \times m_a$$

with m_a = mass of the atmosphere (per unit area) and M_i = molecular weight of the element i, we can find the corresponding volume concentration c_{O_2}:

$$\begin{aligned} c_{O_2} &= \frac{m_{O_2}}{m_a} \frac{\sum c_i M_i}{M_{O_2}} \\ &= \frac{2.353 \times 10^3}{1.017 \times 10^4} \times \frac{28.97}{32} \\ &= 0.2095 \end{aligned}$$

which is the value given in Table 1.1.

2.15 The current rate of consumption of fossil fuels is 7 Gt C per year. Based on the data in Table 2.3, how long would it take to deplete the entire fossil fuel reservoir of fossil fuels (a) if consumption continues at the present rate and (b) if consuption increases at a rate of 1% per year over the next century and remains constant thereafter.

Solution: The quantity of carbon in fossil fuels given in Table 2.3 is m =10 kg m^{-2} (averaged over the Earth's surface). The total mass of carbon in fossil fuels over the Earth is $m_{tot} = m \times 4\pi R_E^2$ =5100 Gt

(a) if consumption continues at the present rate r_o (=7 Gt C per year), the time required for depletion t_d is:

$$\begin{aligned} t_d &= \frac{m_{tot}}{r_o} = \frac{10 \times 4\pi \times (6.37)^2 \times 10^{12}}{7 \times 10^{12}} \\ &= \frac{4\pi \times (6.37)^2}{7} \times 10 = 728 \text{ years.} \end{aligned}$$

(b) if consumption increases at a rate of 1% per year over the next century:

$$r = r_o e^{0.01t}$$

Substituting $t = 100$, we find the rate of consumption at t = 100 year: $r_{100} = r_o e = 19$ Gt per year. The total consumption until year 100 is:

$$\begin{aligned} C_{100} &= \int_0^{100} r_o e^{0.01t} dt \\ &= \frac{r_o}{0.01} \times \left[e^{0.01t}\right]_0^{100} \\ & \quad r_o \times 100 \times (e-1) \\ &= 7 \times 100 \times (e-1) \\ &= 1203 \text{ Gt.} \end{aligned}$$

The carbon left after 100 years is $m_{tot} - C_{100}$; it is depleted at the constant rate r_{100} in a time

$$\begin{aligned} t &= \frac{m_{tot} - C_{100}}{r_{100}} \\ &= 205 \text{ years} \end{aligned}$$

The total time of depletion is $t_d = 100 + 205 = 305$ years.

2.16 If all the carbon in the fossil fuel reservoir in Table 2.3 were consumed, and if half of it remained in the atmosphere in the form of CO_2, by what proportion would the atmospheric concentration of CO_2 increase relative to current values? By what proportions would the atmospheric O_2 concentration decrease?

Solution: Half of the quantity of carbon in fossil fuels is 5 kg m^{-2}, which is added to the present quantity of carbon in atmospheric CO_2 (1.6 kg m^{-2}).

Using the relation

$$m_C = \frac{c_{CO_2} \times M_C}{\sum c_i M_i} \times m_a$$

with m_a = mass of the atmosphere (per unit area) and M_i = molecular weight of the element i, we can find the corresponding volume concentration c_{O_2}:

$$\begin{aligned} c_{CO_2} &= \frac{m_C}{m_a} \frac{\sum c_i M_i}{M_C} \\ &= \frac{(5 + 1.6)}{1.017 \times 10^4} \times \frac{28.94}{12} \\ &= 1.57 \times 10^{-3} = 1570 \text{ ppmv} \end{aligned}$$

The increase in CO_2 implies a decrease in O_2 (for each molecule of C, one molecule of O_2 is needed to form one molecule of CO_2). Therefore (1570 – 380) ppmv of oxygen would disappear.

The decrease of O_2 concentration would be of order: $\frac{(1570-380)\times 10^{-6}}{0.2095}$ ~0.6%.

2.17 Using the Tables presented in this chapter, compare the mass of water lost from the hydrosphere, due to the escape of hydrogen to space over the lifetime of the earth, with the mass of water currently residing in the oceans.

Solution: From Table 2.4 it is evident that the mass of oxygen liberated over the lifetime of the earth is at least 500 10^3 kg m^{-2}. Since H_2O is 16/18 oxygen by mass, a nearly equal mass of water must have escaped from the Earth system. For comparison, the mass per unit area of the oceans, 2,700 10^3 kg m^{-2}, is over 5 times as large.

2.18 The half life of ^{14}C is 5,730 years. If c_0 is the ambient concentration of atmospheric ^{14}C, estimate the abundance of ^{14}C remaining in a 50,000 year old sample.

Solution: A decay in time is represented by an exponential relation:

$$c = c_o e^{-\frac{t}{\tau}}$$

where c_o is the initial concentration, τ is the $e-folding$ time (time to decrease by a factor e). To find the relation between e-folding time and *half-life* (t_{hl}), we impose:

$$\begin{aligned} \frac{c}{c_o} &= \frac{1}{2} = e^{-\frac{t_{hl}}{\tau}} \\ t_{hl} &= \ln(2) \times \tau \end{aligned}$$

Therefore in this example $\tau = t_{hl}/\ln(2) = 8267$ years. The concentration after 50,000 years is

$$\begin{aligned} c &= c_o e^{-\frac{t}{\tau}} \\ &= c_o e^{-\frac{50,000}{8267}}; \\ \frac{c}{c_o} &= 2.36 \times 10^{-3} \end{aligned}$$

Alternatively, we could solve this problem using the equivalent relation:

$$c = c_o 2^{-\frac{t}{t_{hl}}}$$

2.19 If all the carbon in the inorganic and organic sedimentary rock reservoirs in Table 2.3 were in the atmosphere instead, in the form of CO_2, together with the atmosphere's present constituents, what would the mean surface pressure be? What would be the volume concentration of N_2?

Solution: The quantity of carbon in sedimentary rocks is: m_c =100,000 kg m^{-2}. If this carbon were in the atmosphere instead, the atmospheric mass would be dominated by carbon (the present mass is only: m_a =1.017 $\times 10^4$ kg m^{-2}). We need to include the mass of oxygen to find the mass of CO_2 m_{CO_2} (for one molecule of C, one molecule of CO_2 is formed)

$$m_{co_2} = m_c \times \frac{M_{CO_2}}{M_c}$$

The new atmospheric mass would be $m_{new} = m_a + m_{co_2} = 1.017 \times 10^4 + 100,000 \times 44/12 = 3.77 \times 10^5$ kg m^{-2} (~37 times m_a). The new atmospheric pressure would be $p_{new} = m_{new} g \sim 37 \times 10^5$ Pa.

The mass of N_2 is constant:

$$\begin{aligned} m_{_{N_2}} &= \frac{c_{_{N_2}} \times M_{_{N_2}}}{\sum c_i M_i} \times m_a \\ \frac{c_{_{N_2}}}{\sum c_i M_i} \times m_a &= \frac{(c_{_{N_2}})_{new} \mu}{(\sum c_i M_i)_{new}} \times m_{new} \end{aligned}$$

Therefore the new fractional concentration of N_2 would be:

$$(c_{N_2})_{new} = c_{N_2} \times \frac{m_a}{m_{new}} \times \frac{(\sum c_i M_i)_{new}}{\sum c_i M_i}$$
$$\sim 0.78 \times \frac{1}{37} \times \frac{44}{29} \sim 3\%$$

(assuming that the new apparent molecular weight of the atmosphere $\sim M_{CO_2}$).

2.20 Averaged over the atmosphere as a whole, the drawdown of atmospheric carbon dioxide due to photosynthesis in the terrestrial biosphere during the growing season in the northern hemisphere is ~4 ppmv, or about 1% of the annual mean atmospheric concentration. Estimate the area-averaged mass per unit area of carbon incorporated into leafy plants in the extratropical northern hemisphere continents. Assume that these plants occupy roughly 15% of the area of the earth's surface.

Solution: From Table 2.3 the mass of atmospheric carbon, averaged over the area of the surface of the Earth 's 1.6 kg m^{-2}. If the storage of carbon in leafy plants is 1% of this amount and if in is concentrated in 15% of the area of the Earth, then the mass per unit area is on the order of 0.1 kg m^{-2}.

2.21 At the time of the Last Glacial Maximum (LGM), global sea-level was ~125 m lower than it is today. Assuming that the lower sea-level was due to the larger storage of water in the northern hemisphere continental ice sheets, compare the mass of the northern hemisphere ice sheets at the time of the LGM with the current mass of the Antarctic ice sheet.

Solution: In Exercise 2.2 in the text it is shown that a if the Greenland ice cap were to entirely melt, global sea-level would rise by 7 m. In Table 2.2 it is shown that the current mass of the Greenland ice sheet is 5×10^3 kg m^{-2} and the current mass of the Greenland and Antarctic ice sheets is 58×10^3 kg m^{-2}. Hence the mass of water stored in today's ice sheets is is sufficient to cause a 7 m $\times (58/5) = 81$ m rise in sea-level if it were all to melt. (This rough estimate ignores the small increase in the area of the oceans as sea-level rises.) If all of the water stored in the continental ice sheets at the time of the LGM were to melt, sea-level would rise by $81+125 = 206$ m. Hence the storage of water in the continental ice sheets at the time of the LGM was $206/81 = 2.54$ times as large as it is today.