WEATHER SIGNALS
Chapter 4
(Focus is on weather signals or echoes from radar resolution volumes filled with countless discrete scatterers---rain, insects, perturbations in atmospheric refractive index, etc.)
Weather Signal Characteristics

- Large dynamic range (100 million!)
- Signals are semi coherent
- Numerous scatterers in the radar resolution volume

The choice instrument for weather surveillance is the pulsed polarimetric Doppler weather radar.

Extracting information (i.e., fields of echo H, V power, Doppler velocity, and correlation of H, V echoes) involves processing of echoes that randomly fluctuate.
Echoes (I or Q) from Distributed Scatterers (Fig. 4.1)

\[ \tau_c(\tau_s) \approx \tau_t \]

(\(\tau_t = \) transmitted pulse width)
Resolution Volumes

9.6 km

600 m

16 RANGE GATES
BEAM WIDTH 3°
Range resolution 150 m

Lightning detector

10 cm DOPPLER RADAR

SLOW ANTENNA
FAST ANTENNA
TIME CODE

LOG VIDEO
I, Q, TIME REF

DIGITAL PROCESSING & RECORDING

ANALOG RECORDING
Echo samples from 16 Resolution Volumes (Fig. 4.3)

\[ T_s = 768 \text{ ms} \]
Gate 12 Signal Spectrum (Fig. 8.34)

\[ MT_s = 128 \times 0.768 \text{ ms} = 98.3 \text{ ms} \]

12 spectra are averaged
Repeat of Fig. 4.3

\[ \tau_c(mT_s) \approx 2-4 \text{ ms} \]
Statistics of Weather Signals

I and Q are uncorrelated zero mean random variables with Gaussian probability density function (pdf)

P has an exponential pdf

β has a uniform pdf

Amplitude |V| has a Rayleigh pdf

Thanks to Dr. Sebastian Torres
Weather Echo Statistics (Fig. 4.4)

Random processes \( I_n \) and \( Q_n \) are correlated.

Uncorrelated random variables (Gaussian distribution)
Weighting Functions for Scatterers and the Weather Radar Equation

\[
\overline{P_r(r_o)} \approx \frac{P_t g^2 \lambda^2 \eta(r_o)}{(4\pi)^3 r_o^2 \lambda^2 (r_o)} \int \left| W_s(r) \right|^2 \, dr \int d\theta \int f^4(\theta, \varphi) \sin \theta d\varphi
\]

(4.12)

where \( \eta \) is the reflectivity (i.e., backscatter cross section per unit volume--assumed spatially uniform):

\[
\eta(r) = \int_0^\infty \sigma_b(D) N(D, r) dD \quad (m^2 m^{-3}) \quad (4.10)
\]

and \( N(D, r) \equiv \text{size distribution (m}^{-3} \text{mm}^{-1}) \)

\[
\left| W_s(r) \right|^2 \equiv \text{the range weighting function;}
\]

\( f^4(\theta, \varphi) \equiv \text{the angular weighting function.} \)
Drop Size Distributions (Fig. 8.3b)

\[ N(D) = N_0 \exp(-\Lambda D) \quad \text{(EXP.DSD)} \]

\[ N_0 = 8 \times 10^3 \, \text{m}^{-3} \, \text{mm}^{-1} \quad \text{(MP DSD)} \]

(From drop sizes between 1 and 3 mm)

Marshall and Palmer (1948)

Laws and Parsons (1943)
The Angular Weighting Function

\[ \int_0^{2\pi} \int_0^{\pi} f^4(\theta,\phi)\sin\theta d\theta d\phi = \pi \theta_1^2 / 8 \ln 2 \]

for circularly symmetric Gaussian pattern
The Measured Range Weighting Function for two Receiver Bandwidths (Fig. 4.6)
Range Weighting Function for Echoes Samples at Range Time $\tau_s$

(Fig. 4.7)

\[ |W(r)|^2 \]

\[ c\tau_s / 2 \]

\[ c\tau_r / 2 \]

\[ r_0 \]

\[ r = 0 \]
The Resolution Volume $V_6$

Fig. 5.11

Angular weighting function

Range weighting function

$V + dV = \text{const}$

$\gamma(r_i)$

$V(r_i) = \text{const}$

$\mathbf{r}_0$, $\phi_0$, $\theta_0$

$\mathbf{r}_1$, $\phi_1$, $\theta_1$
If receiver frequency response is matched to the spectrum of the transmitted pulse (an ideal matched filter receiver), some echo power will be lost. This is called the finite bandwidth receiver loss $L_r$. For an ideal matched filter $L_r = 1.8$ dB.
Receiver Loss Factor (Fig. 4.8) for a Gaussian receiver response and rectangular pulse (in general a matched condition is when $B_6 \tau = 1$)

$$L_r = 1.8 \text{ dB for an ideal matched receiver}$$

$B_6 = 6 \text{ dB bandwidth of the receiver’s frequency response.}$

$\tau = \text{transmitted pulse width}$

$\text{Eq. (4.28)}$
Reflectivity Factor $Z$
(Spherical scatterers; Rayleigh condition: $D \leq \lambda/16$)

$$\eta(r) = \frac{\pi^5}{\lambda^4} \left| K_m \right|^2 Z(r) \quad (4.31)$$

where

$$Z(r) \equiv \frac{1}{\Delta V} \sum_i D_i^6 = \int_0^\infty N(D, r) D^6 dD \quad (4.32)$$

$$\eta(r) = \frac{\pi^5}{\lambda^4} \left| K_w \right|^2 Z_e(r) \quad (4.33)$$

for water drops : $\left| K_w \right|^2 \approx 0.93$ independent of $T (^{\circ}C)$;
for ice particles : $\left| K_i \right|^2 \approx 0.16$ dependent on $T$ and ice density.
Reflectivity Factor of Spheroids
(Horizontally Polarized Waves)

\[ Z_h = \frac{\lambda^4 N_o}{\pi^5 |K|^2} \int_0^\infty \int_0^\xi \int_{\xi}^{\infty} p[D_e, e, \xi] \sigma_h[D_e, e, \xi] dD_e d\xi de \]

\[ p[D_e, e, \xi] = \text{probability density} \]
\[ \sigma_h[D_e, e, \xi] = \text{backscatter cross section for H pol.} \]
\[ D_e = \text{equivalent volume diameter} \]
\[ e = \text{eccentricity of the spheroid scatterer} \]
\[ \xi = \text{angle between the symmetry axis and the electric field direction} \]
\[ N_0 = \text{the number density per unit diameter (m}^{-4}) \]
Differential Reflectivity

in dB units:
\[ Z_{DR}(dB) = Z_h(dBZ) - Z_v(dBZ) \]

in linear units:
\[ Z_{dr} = \frac{Z_h(mm^6m^{-3})}{Z_v(mm^6m^{-3})} \]

- is independent of drop concentration \( N_0 \)
- depends on the shape of scatterers
Shapes of raindrops falling in still air and experiencing drag force deformation.

$D_e$ is the equivalent diameter of a spherical drop. $Z_{DR}$ (dB) is the differential reflectivity in decibels (Rayleigh condition is assumed). Adapted from Pruppacher and Beard (1970)
The Weather Radar Equation

A form of the weather radar equation for echo power from rain is:

\[
E[P(r_0)](\text{mW}) = \frac{\pi^{5/10} P_t(W) g_s \tau(\mu s) \theta_{1/2}^2 (\text{deg.}) | K_w |^2 Z_w (\text{mm}^{-3})}{6.75 \times 2^{14} (\ln 2) r_0^2 (\text{km}) \lambda^2 (\text{cm}) l_r^2}\]

(4.35)

\[
E[P(r_0)] = \text{Expected peak weather signal power in milliwatts;}
\]

\[
P_t = \text{Peak transmitted pulse power (typically 500 kW)}
\]

\[
g_s = \text{net power gain of the echo in going from the antenna to the radar output.} \; \tau = \text{pulse width}
\]

\[
\theta_{1/2} = \text{one-way half-power beamwidth;} \; | K_w |^2 = \text{dielectric factor of water}
\]

\[
Z_w = \text{reflectivity factor for water spheres;} \; r_0 = \text{range (in km) to the center of the resolution volume V}_6
\]

\[
l = \text{one-way loss factor (a number } \geq 1) \text{ incurred for propagation through a rain filled atmosphere.}
\]

\[
l_r = \text{loss factor due to the finite bandwidth of the receiver;} \; \lambda = \text{wavelength of the transmitted radiation}
\]
Acquisition, Processing and Display of Weather radar data

Product displays (e.g., CAPPI, etc.)

Volume Coverage Pattern

Azimuthal Scan (constant elevation)

Data radial

Sample-time

$\tau_s$

$\tau_s$

Time series of $M$ power samples

Base Data: $P_h$, $P_v$, $v$, $SW$, etc.

Radial of reflectivity factor $Z$

Radial of reflectivity factor $Z$

Range $r = c\tau_s/2$

(M power samples are processed to produce one $Z$ estimate at $r$)
WSR-88D Thresholding Data Fields based on Signal to Noise Ratios

Locations with non-significant powers are censored:

- Non significant returns have a SNR below a user-defined threshold

- The system allows a different threshold for each spectral moment
SNR > -3 dB
VELOCITY (m/s)
VCP48, DATE:04:03:04, TIME:20:38:33, CUT # 2 (0.44°)

150 km
100 km
50 km

\( v(R_h) \), \( SNR > 3.5 \, dB \)
Weather Echo Power vs Range (WSR-88D)

\[ P_r = P_n \approx 6 \times 10^{-15} \text{ Watts} \]