8/25/20

x-eqn: $\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$

y-eqn: $\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$

} neglect $\vec{F}_{fric}$

z-eqn: $\frac{Dw}{Dt} - B = \frac{g(\theta - \bar{\theta})}{\bar{\theta}}$ non hydrostatic

define $m = u - fy$ $\left(m_g = u_g - fy\right)$

$$\frac{Dm}{Dt} = \frac{Du}{Dt} - f\underbrace{\frac{Dy}{Dt}}_{v} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

if $\frac{\partial}{\partial x} = 0$ ("symmetric") $\Rightarrow \frac{Dm}{Dt} = 0$

$\Rightarrow$ m is conserved for frictionless, symmetric flow

recall $u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \Rightarrow -\frac{1}{\rho}\frac{\partial p}{\partial y} = f u_g$

so, $\frac{Dv}{Dt} = f u_g - f u = -f(u - u_g)$

but $m - m_g = u - u_g$

so, $\frac{Dv}{Dt} = -f(m - m_g)$

$\therefore$ 2 governing eqns:

$$\begin{cases} \frac{Dv}{Dt} = -f(m - m_g) \\ \frac{Dw}{Dt} = \frac{g}{\bar{\theta}}(\theta - \bar{\theta}) \end{cases}$$

tube, env, env, tube