Gravity Currents

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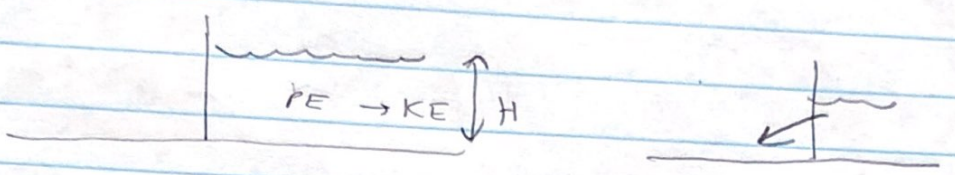
dam break problem



$$\Delta KE \rightarrow -(\Delta PE)$$

$$\frac{1}{2}U^2 = \frac{gH}{2} \quad \Rightarrow U = \sqrt{gH}$$

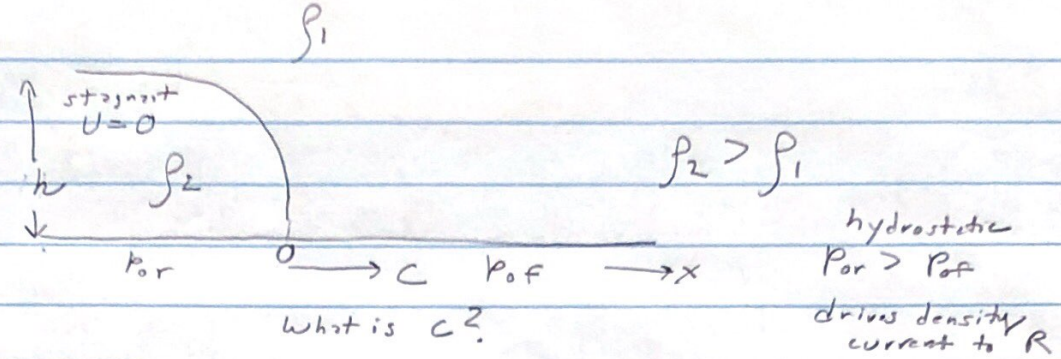
$\sqrt{10 ms^{-2}(10m)} = 10 ms^{-1}$

now, add viscosity — slow down, affect shape of current

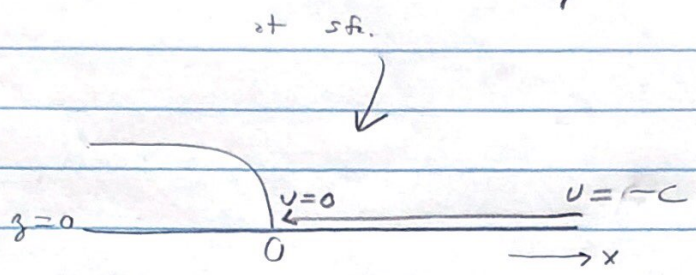
mention Great Molasses Flood of
in Boston 1919 killed 21 people injured 150

now, replace air above dam by only slightly less dense fluid

9/20/20



in reference frame of density current:



steady; 2-D symmetry; $w=0$ at $z=0$

$$\cancel{\frac{\partial U}{\partial t}} + U\frac{\partial U}{\partial x} + V\cancel{\frac{\partial U}{\partial y}} + w\cancel{\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

ignore friction (gulp!)

$$\frac{\partial}{\partial x}\left(\frac{1}{2}U^2\right) + \frac{1}{\partial x}\left(\frac{p'}{\rho}\right) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}U^2 + \frac{p'}{\rho}\right) = 0$$

$$\Rightarrow \frac{1}{2}U^2 + \frac{p'}{\rho} \text{ conserved} \qquad \rho = \rho_1$$

$$\frac{1}{2}(-C)^2 + 0 + \frac{\cancel{p'(x=\infty)}^{\,0}}{\rho_1} - \frac{p'(x=0)}{\rho_1} = 0$$

at $x=\infty$; at $x=0$ ↑ just inside density current

$$\frac{1}{2}c^2 = \frac{p'(x=0)}{\rho_1}$$

$$\frac{Dw}{Dt} = 0 = -\frac{1}{\bar{\rho}}\frac{dp'}{dz} + B$$

$$B = g\left(\frac{\bar{\rho}-\rho}{\bar{\rho}}\right)$$

at $x=0^-$, $\frac{1}{\bar{\rho}}\frac{dp'}{dz} = g\left(\frac{\rho_1-\rho_2}{\rho_1}\right)$

for $z \le h$

$$\int_0^h \frac{1}{\bar{\rho}}\frac{dp'}{dz}\,dz = \int_0^h g\left(\frac{\rho_1-\rho_2}{\rho_1}\right)dz$$

$$\frac{1}{\bar{\rho}}\left(p'(z=h) - p'(z=0)\right) = g\left(\frac{\rho_1-\rho_2}{\rho_1}\right)h$$

set $=0$ (justify:

air at $z=h$ doesn't know about density current below)

$$\frac{p'(z=0)}{\bar{\rho}} = g\left(\frac{\rho_2-\rho_1}{\rho_1}\right)h$$

$$\frac{p'(z=0)}{\rho_1}_{x=0^-} = g\left(\frac{\rho_2-\rho_1}{\rho_1}\right)h$$

so, $\frac{1}{2}c^2 = gh\left(\frac{\rho_2-\rho_1}{\rho_1}\right)$

$$c = \sqrt{2gh\left(\frac{\rho_2-\rho_1}{\rho_1}\right)}$$

add friction, C is slower

$$c = K\sqrt{2gh\frac{\rho_2 - \rho_1}{\rho_1}} \qquad K < 1$$

$g < h$

$+ (0.62\,\vec{U})$

→ add basic current
→ add vertical shear

} covered in convection class

from earlier, $c = \sqrt{2gh\left(\frac{\theta - \theta_2}{\theta_1}\right)}$

replace ρ by θ using ideal gas law or defn. of $\theta \sim \theta(p, T)$